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A Novel Conformable Triple Elzaki Transform Decomposition Method for Nonlinear Fractional Telegraph Equations

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Abstract
This paper introduces a novel hybrid analytical approach, namely the Conformable Triple Elzaki Decomposition Method (CTEDM), for obtaining exact solutions of two-dimensional nonlinear conformable fractional telegraph equations. The proposed method combines the Conformable Triple Elzaki Transform (CTET) with Adomian's decomposition method to effectively handle both linear and nonlinear components of the problem. Several fundamental properties and theorems of the CTET are established to support the analytical framework. The linear part of the governing equation is solved using the CTET, while the nonlinear terms are efficiently treated via Adomian polynomials. Remarkably, the proposed method requires only a minimal number of iterations to obtain exact or near-exact solutions. To demonstrate the accuracy and applicability of the method, benchmark problems from mathematical physics are considered, and the obtained results show excellent agreement with expected analytical behavior. Compared to existing techniques such as Laplace transform, Reduced Differential Transform Method (RDTM), and classical Adomian decomposition approaches, the proposed CTEDM offers faster convergence, reduced computational complexity, and improved analytical tractability. The results confirm that the method is efficient, reliable, and capable of producing highly accurate solutions with minimal computational effort. Furthermore, the proposed approach has strong potential for extension to a wide class of nonlinear fractional partial differential equations arising in applied mathematics and physics.

Keywords: Conformable Fractional Derivative; Conformable Triple Elzaki Transform; Nonlinear Fractional Telegraph Equation; Adomian's Polynomial; Conformable Triple Elzaki Decomposition Method.

1. Introduction

Fractional calculus is a field of mathematical analysis that generalizes classical differentiation and integration to non-integer orders. It has become a powerful tool for modeling memory and hereditary properties in many real-world problems. The concept of fractional derivatives was first introduced in the correspondence between L'Hôpital and Leibniz, where the idea of a fractional (half-order) derivative was first discussed [1], [2]. In particular, fractional partial differential equations (FPDEs) have been widely applied in diverse fields, including fluid dynamics [3], electrical transmission lines [4], biology [5], and viscoelasticity, anomalous diffusion and control theory [2]. Consequently, a growing number of researchers have devoted their efforts to finding solutions to FPDEs over the past few decades [2], [6]–[10]. Singh [5] analyzed a fractional blood alcohol concentration model incorporating a composite fractional derivative, demonstrating its applicability in modeling complex biological processes with memory effects. Throughout the centuries, various definitions of fractional derivatives have been introduced [11], [12], including the Riemann–Liouville [2], Caputo [2], Caputo–Fabrizio [13], Grünwald–Letnikov [14], and Hadamard [15] formulations. However, these operators do not always retain fundamental properties of classical calculus, such as the derivative of a constant being zero, along

with the product and chain rules. For example, the Riemann–Liouville derivative of a constant is not zero. These fractional derivatives share only one property of the classical derivative, namely linearity. To overcome these limitations, Khalil et al. [16] proposed the Conformable Fractional Derivative (CFD), which retains many fundamental properties of classical differentiation and provides a more convenient framework for both analytical and numerical analysis. The definition is given as follows:

For $\alpha \in \mathbb{R}$, the CFD $D_\alpha^\delta f(x)$ of a real function $f:[\alpha,\infty) \to \mathbb{R}, \delta \in (0,1]$ is defined as

$$D_\alpha^\delta f(x) = \lim_{h\to 0} \frac{f\left(x+h(x-\alpha)^{1-\delta}\right)-f(x)}{h}, \qquad x > \alpha,\ \delta \in (0,1] \tag{1}$$

f is called δ-differentiable at $x > \alpha$ provided the limit exists.

This fractional derivative satisfies all basic rules of classical derivative. Due to this, it is easier to use and more intuitive for solving differential equations, particularly in modeling real-world phenomena. However, Tarasov [17] showed that a derivative satisfying the standard Leibniz rule must be of integer order; consequently, the conformable derivative is a local operator and does not capture memory effects in the sense of nonlocal fractional operators such as the Riemann–Liouville and Caputo derivatives. Nevertheless, its simple calculus makes it convenient for analytical methods such as the one developed in this paper. Abdeljawad [18] further developed the conformable fractional calculus, establishing fractional versions of the chain rule, exponential functions, Gronwall's inequality, integration by parts, Taylor power series expansions and Laplace transforms. In 2021, Kajouni et al. [19] introduced a new form of the CFD and demonstrated its effectiveness through various applications, providing a more flexible framework for modeling fractional-order systems. Several authors employed various techniques like Reduced Differential Transform method [20], [21], Double Laplace-Sumudu Transform [22]–[24], Double Sumudu-Elzaki transform [25], [26], Double Laplace Transform [27], Elzaki Transform Homotopy Perturbation Method (HPM) [28] and Double Laplace-Sawi Transform [29] in the CFD senses to obtain the analytical solutions of different types of linear and non-linear fractional partial differential equations and integral equations. Tayyan and Sakka [30] explored the application of Lie group theory to conformable fractional partial differential equations, establishing new solution techniques through symmetry reductions. In another study, Abul-Ez et al. [31] introduced conformable fractional Legendre polynomials and investigated their convergence properties, along with practical applications in fractional calculus. Recently, dynamical-system models have also been used to study intervention strategies for epidemics such as COVID-19 [32].

The telegraph equation was originally developed in the 19th century as a mathematical model for electrical transmission lines. It has subsequently been commonly employed in characterizing signal propagation over transmission systems, as well as numerous wave phenomena. Due to this, the telegraph equation is in many aspects a better formulation of some reaction–diffusion processes than the classical diffusion equation. Nevertheless, various physical systems show memory and hereditary behavior which cannot be treated satisfactorily with integer-order models. To address this limitation, fractional forms of the telegraph equation have been proposed based on types of operators such as Riemann–Liouville and Caputo derivatives. One of the major developments in this direction was the conformable fractional derivative introduced by Khalil et al. [16], which preserves the fundamental features of classical calculus and broadens the use of analytical and semi-analytical techniques. As a result, the conformable fractional telegraph equation emerged as an effective and physically meaningful generalization of classical model. Many analytical and numerical approaches have been suggested to solve telegraph equations in recent years [33]. Some methods, including the transformation-based methods such as Laplace transformation have successfully been utilized to obtain exact solutions [34] but recently, some other integral

transformations especially Double Laplace transform [27], [35] and Sumudu transform [36] have also greatly simplified the solution prediction process. Similarly, the conformable double convolution Laplace transform used to numerically solve fractional telegraph equations efficiently and accurately [35]. Moreover, hybrid and iterative methods including Conformable Triple Laplace Transform Iterative Method (CTLTM) [37], provide rapid convergent series solutions. Semi-analytical methods, such as the Reduced Differential Transform Method (RDTM), have been widely used for solving linear and nonlinear fractional models. For instance, Srivastava et al. [38] used RDTM with the Caputo time-fractional telegraph equation, whereas Deresse [39] showed the method is effective for nonlinear conformable problems. More recently, Chu et al. [40] developed a local time-fractional RDTM for improved handling of complex dynamical systems. Moreover, there has been effective implementation of numerical and analytical methods similar to the Fourier method as well as theta-approach for conformable fractional telegraph equations [41], [42]. Although there are various transform techniques, the Elzaki transform has drawn increasing attention in finding precise and approximate solutions of fractional differential equations [43]. The above have led to several modified variants, e.g., Conformable Elzaki Transform (CET) [44] and Conformable Double Elzaki Transform (CDET) [45], and the fractional triple Elzaki transform [46], which have proven effective in solving nonlinear fractional models. However, nonlinear problems often lead to iterative procedures or increased computational complexity.

Among these studies, the works closest to the present one differ in the transform used, the number of variables, or the treatment of the nonlinearity. Kumar [34] and Osman et al. [35] applied Laplace-type transforms to fractional telegraph equations, mainly in the linear case. Anaç [43] combined the single-variable conformable Elzaki transform with a decomposition method, while the conformable Elzaki transform [44] and the conformable double Elzaki transform [45] were developed for problems in one and two variables, respectively. Mousa [46] introduced the fractional triple Elzaki transform and established its properties but did not apply it to nonlinear partial differential equations. The closest work is that of Deresse [37], who solved two-dimensional nonlinear telegraph equations with the conformable triple Laplace transform iterative method. Since the Elzaki and Laplace transforms are related by $H(u) = u\,F\left(\frac{1}{u}\right)$, both approaches are equivalent in principle; however, the Elzaki transform gives simpler expressions for polynomial and exponential data, which simplifies the algebra of the inverse step. To the best of our knowledge, the conformable triple Elzaki transform has not previously been combined with the Adomian decomposition method for two-dimensional nonlinear conformable telegraph equations, which is the gap addressed in this paper.

Taking these problems into account, this study presents a novel hybrid analytical method called the Conformable Triple Elzaki Decomposition Method (CTEDM). This method combines the Conformable Triple Elzaki Transform (CTET) with the Adomian Decomposition Method (ADM). George Adomian [47] came up with this method, which is termed the ADM, and it has been used in many ways with nonlinear differential equations. The convergence and effectiveness of the ADM have been studied in [48], [49]. In the last few years, numerous hybrid strategies that use transform methods like Laplace, Sumudu, and Elzaki transforms have been effectively used with ADM to improve the solution's efficiency. In [50], a conformable Sumudu decomposition method was introduced, while in [51], Hassan and Elzaki proposed the Double Elzaki Transform Decomposition Method as an approach for solving nonlinear partial differential equations. This method works even when the components are not linear, and it does not need linearization or discretization. It uses a method of breaking down the nonlinear term that gives correct or very accurate solutions that converge quickly and do not require as much computing power [47]–[51]. For the problems considered in this paper, the proposed CTEDM yields the exact solution in a single iteration. To highlight the features of the proposed method, a qualitative comparison with existing techniques is presented in Table 1.

Table 1. Qualitative comparison of existing methods and the proposed CTEDM

Method	Type	Nonlinearity Handling	Convergence	Computational Effort	Accuracy	Remarks
Laplace Transform Method	Analytical	Limited	Moderate	Moderate	Good	Effective for linear problems; inversion can be difficult
Sumudu Transform Method	Analytical	Limited	Moderate	Moderate	Good	Simpler inversion than Laplace; limited nonlinear capability
ADM	Semi-analytical	Good	Slow	High	Moderate	Handles nonlinear terms but requires many iterations
HPM	Semi-analytical	Good	Moderate	Moderate	Good	Depends on initial approximation
RDTM	Semi-analytical	Good	Moderate	Moderate	Good	Provides approximate series solutions
Double Elzaki Transform Methods	Analytical	Moderate	Moderate	Moderate	Good	Effective hybrid transform approach
Proposed CTET+ ADM (CTEDM)	Hybrid	Strong	Fast	Low	High	Provides exact or near-exact solutions with minimal computational effort

The proposed CTEDM has been employed to solve the two-dimensional nonlinear conformable telegraph equation (for $\delta = \xi = \zeta = 1$, see [33])

$$\frac{\partial^{2\zeta} h}{\partial \psi^{2\zeta}} + p\frac{\partial^{\zeta} h}{\partial \psi^{\zeta}} + qN\left(h\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right)\right) = r\left(\frac{\partial^{2\delta} h}{\partial \phi^{2\delta}} + \frac{\partial^{2\xi} h}{\partial \varphi^{2\xi}}\right) + g\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right),$$

$$\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta} > 0 \; and \; \delta, \xi, \zeta \in (0,1] \tag{2}$$

with initial conditions $h\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, 0\right) = a_1\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}\right), \frac{\partial^{\zeta}}{\partial \psi^{\zeta}} h\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, 0\right) = a_2\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}\right)$ (3)

and Cauchy type boundary conditions

$$h\left(0, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right) = b_1\left(\frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right), \frac{\partial^{\delta}}{\partial \phi^{\delta}} h\left(0, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right) = b_2\left(\frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right) \tag{4}$$

$$h\left(\frac{\phi^{\delta}}{\delta}, 0, \frac{\psi^{\zeta}}{\zeta}\right) = b_3\left(\frac{\phi^{\delta}}{\delta}, \frac{\psi^{\zeta}}{\zeta}\right), \frac{\partial^{\xi}}{\partial \varphi^{\xi}} h\left(\frac{\phi^{\delta}}{\delta}, 0, \frac{\psi^{\zeta}}{\zeta}\right) = b_4\left(\frac{\phi^{\delta}}{\delta}, \frac{\psi^{\zeta}}{\zeta}\right) \tag{5}$$

where p, q and r are known real constants; $h\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right)$ denotes either the voltage or current through the two-dimensional conductor at position (ϕ, φ) at time ψ; $\delta, \xi, \zeta \in (0,1]$ are the fractional orders; N is the general nonlinear term and $g\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right)$ is the source term.

2. Preliminaries

2.1. Conformable Fractional Derivative

Definition 1: Caputo fractional derivative of $\delta > 0$ order of a function $f(t)$ is expressed in [2] by

$$D_t^{\delta} f(t) = \frac{1}{\Gamma(m-\delta)} \int_0^t (t-\xi)^{m-\delta-1} \frac{d^m f}{d\xi^m} d\xi, \quad m-1 < \delta \le m, \quad m \in N$$

Definition 2: Define $h : \mathbb{R}^+ \to \mathbb{R}$, then the conformable fractional derivative of $h\left(\frac{\phi^{\delta}}{\delta}\right)$ of order δ is defined as

$$\frac{d^{\delta}}{d\phi^{\delta}} h\left(\frac{\phi^{\delta}}{\delta}\right) = \lim_{\varepsilon \to 0} \frac{h\left(\frac{\phi^{\delta}}{\delta} + \varepsilon \phi^{1-\delta}\right) - h\left(\frac{\phi^{\delta}}{\delta}\right)}{\varepsilon}, \frac{\phi^{\delta}}{\delta} > 0, 0 < \delta \le 1. \tag{6}$$

Definition 3: See [52] Suppose $h: (0,\infty) \times (0,\infty) \times (0,\infty) \to \mathbb{R}$, thus the conformable partial fractional derivatives of $h\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right)$ of orders δ, ξ and ζ respectively are expressed by

$$\partial_{\phi}^{\delta} h = \frac{\partial^{\delta}}{\partial \phi^{\delta}} h\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right) = \lim_{\varepsilon \to 0} \frac{h\left(\frac{\phi^{\delta}}{\delta} + \varepsilon \phi^{1-\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right) - h\left(\frac{\phi^{\delta}}{\delta}, \frac{\varphi^{\xi}}{\xi}, \frac{\psi^{\zeta}}{\zeta}\right)}{\varepsilon}, \tag{7}$$

$$\partial_{\varphi}^{\xi}h=\frac{\partial^{\xi}}{\partial\varphi^{\xi}}\,h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\lim_{\varepsilon\to0}\frac{h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi}+\varepsilon\varphi^{1-\xi},\frac{\psi^{\zeta}}{\zeta}\right)-h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)}{\varepsilon},\tag{8}$$

$$\partial_{\psi}^{\zeta}h=\frac{\partial^{\zeta}}{\partial\psi^{\zeta}}\,h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\lim_{\varepsilon\to0}\frac{h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}+\varepsilon\psi^{1-\zeta}\right)-h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)}{\varepsilon},\tag{9}$$

where $0<\delta,\xi,\zeta\leq1$, $\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}>0$ and $\partial_{\phi}^{\delta}h,\partial_{\varphi}^{\xi}h,\partial_{\psi}^{\zeta}h$ are referred to as the fractional partial derivatives of orders δ,ξ and ζ respectively.

Theorem 1: See [52] Suppose $\delta,\xi,\zeta\in(0,1]$ and $h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ be differentiable at a point for $\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}>0$. Then,

$$(1)\ \partial_{\phi}^{\delta}h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\phi^{1-\delta}\left(\frac{\partial}{\partial\phi}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$$

$$(2)\ \partial_{\varphi}^{\xi}h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\left(\frac{\partial^{\xi}}{\partial\varphi^{\xi}}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\varphi^{1-\xi}\left(\frac{\partial}{\partial\varphi}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$$

$$(3)\ \partial_{\psi}^{\zeta}h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\left(\frac{\partial^{\zeta}}{\partial\psi^{\zeta}}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\psi^{1-\zeta}\left(\frac{\partial}{\partial\psi}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$$

Theorem 2: Let $\delta,\xi,\zeta\in(0,1]$, $a,b,c\in\mathbb{R}$ and $p,q,r\in\mathbb{N}$. Then, we have the following properties:

(1) (Linearity Property) $\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)\left(a\,h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)+b\,h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)$

$=a\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)+b\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$

(2) (Product Rule) $\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)(hg)\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=g\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)+$

$h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)g\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$

$$(3)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)(h/g)\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\frac{g\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)-h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)g\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)}{\left(g\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)^{2}}$$

$$(4)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)\left(\left(\frac{\phi^{\delta}}{\delta}\right)^{p},\left(\frac{\varphi^{\xi}}{\xi}\right)^{q},\left(\frac{\psi^{\zeta}}{\zeta}\right)^{r}\right)=p\left(\left(\frac{\phi^{\delta}}{\delta}\right)^{p-1},\left(\frac{\varphi^{\xi}}{\xi}\right)^{q},\left(\frac{\psi^{\zeta}}{\zeta}\right)^{r}\right),$$

$$\left(\frac{\partial^{\xi}}{\partial\varphi^{\xi}}\right)\left(\left(\frac{\phi^{\delta}}{\delta}\right)^{p},\left(\frac{\varphi^{\xi}}{\xi}\right)^{q},\left(\frac{\psi^{\zeta}}{\zeta}\right)^{r}\right)=q\left(\left(\frac{\phi^{\delta}}{\delta}\right)^{p},\left(\frac{\varphi^{\xi}}{\xi}\right)^{q-1},\left(\frac{\psi^{\zeta}}{\zeta}\right)^{r}\right)$$

$$\left(\frac{\partial^{\zeta}}{\partial\psi^{\zeta}}\right)\left(\left(\frac{\phi^{\delta}}{\delta}\right)^{p},\left(\frac{\varphi^{\xi}}{\xi}\right)^{q},\left(\frac{\psi^{\zeta}}{\zeta}\right)^{r}\right)=r\left(\left(\frac{\phi^{\delta}}{\delta}\right)^{p},\left(\frac{\varphi^{\xi}}{\xi}\right)^{q},\left(\frac{\psi^{\zeta}}{\zeta}\right)^{r-1}\right)$$

$$(5)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)\left(e^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}\right)=ae^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}$$

$$\left(\frac{\partial^{\xi}}{\partial\varphi^{\xi}}\right)\left(e^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}\right)=be^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}$$

$$\left(\frac{\partial^{\zeta}}{\partial\psi^{\zeta}}\right)\left(e^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}\right)=ce^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}$$

$$(6)\left(\frac{\partial^{\delta}}{\partial\phi^{\delta}}\right)\left(sin\left(\frac{\phi^{\delta}}{\delta}\right)sin\left(\frac{\varphi^{\xi}}{\xi}\right)\sin\left(\frac{\psi^{\zeta}}{\zeta}\right)\right)=cos\left(\frac{\phi^{\delta}}{\delta}\right)sin\left(\frac{\varphi^{\xi}}{\xi}\right)\sin\left(\frac{\psi^{\zeta}}{\zeta}\right)$$

$$\left(\frac{\partial^{\xi}}{\partial\varphi^{\xi}}\right)\left(cos\left(\frac{\phi^{\delta}}{\delta}\right)cos\left(\frac{\varphi^{\xi}}{\xi}\right)\cos\left(\frac{\psi^{\zeta}}{\zeta}\right)\right)=-cos\left(\frac{\phi^{\delta}}{\delta}\right)sin\left(\frac{\varphi^{\xi}}{\xi}\right)\cos\left(\frac{\psi^{\zeta}}{\zeta}\right).$$

3. Materials and Methods

3.1. Conformable Elzaki Transform

Suppose the function h: $(0,\infty) \to \mathbb{R}$. Then the conformable Elzaki transform (CET) of the exponential of order δ of function $h\left(\frac{\phi^\delta}{\delta}\right)$ may be expressed and denoted as follows:

$$E_\phi^\delta\left[h\left(\frac{\phi^\delta}{\delta}\right)\right] = H(u) = u\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}}\, h\left(\frac{\phi^\delta}{\delta}\right)\phi^{\delta-1}\, d\phi, \qquad \frac{\phi^\delta}{\delta} > 0 \tag{10}$$

where u is the transform variable of $\frac{\phi^\delta}{\delta}$, and the integral is taken with respect to the conformable measure $d_\delta\phi = \phi^{\delta-1}d\phi$.

3.2. Conformable Double Elzaki Transform

Suppose $h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi}\right)$ be a continuous piece-wise exponential order function on $(0,\infty)\times(0,\infty)$. Then the conformable double Elzaki transform (CDET) of $h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi}\right)$ is expressed and denoted as

$$E_\phi^\delta E_\varphi^\xi\, h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi}\right) = H(u,v) = uv\int_0^\infty\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}-\frac{\varphi^\xi}{v\xi}}\, h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi}\right)\phi^{\delta-1}\varphi^{\xi-1}\, d\phi\, d\varphi \tag{11}$$

where $\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi} > 0$ and u, v are transformation variables of $\frac{\phi^\delta}{\delta}, \frac{\varphi^\xi}{\xi}$ respectively.

3.3. Conformable Triple Elzaki Transform

Suppose $h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$ be a continuous piece-wise exponential order function on $(0,\infty)\times(0,\infty)\times(0,\infty)$. So, the conformable triple Elzaki transform (CTET) of $h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$ is defined and denoted as

$$\begin{aligned} E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\, h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right) &= H(u,v,w) \\ &= uvw\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}-\frac{\varphi^\xi}{v\xi}-\frac{\psi^\zeta}{w\zeta}}\, h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\phi^{\delta-1}\varphi^{\xi-1}\psi^{\zeta-1}\, d\phi\, d\varphi\, d\psi \end{aligned} \tag{12}$$

where $\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta} > 0;\ \delta,\xi,\zeta\epsilon\,(0,1]$ and u, v, w are the transformed variables of $\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}$ respectively.

3.4. Conformable Inverse Triple Elzaki Transform

The conformable inverse triple Elzaki transform of the function $h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$ is defined as

$$h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right) = E_\phi^{\delta^{-1}} E_\varphi^{\xi^{-1}} E_\psi^{\zeta^{-1}}[H(u,v,w)]$$

$$= \frac{1}{(2\pi i)^3}\int_{c_1-i\infty}^{c_1+i\infty}\int_{c_2-i\infty}^{c_2+i\infty}\int_{c_3-i\infty}^{c_3+i\infty} e^{s_1\frac{\phi^\delta}{\delta}+s_2\frac{\varphi^\xi}{\xi}+s_3\frac{\psi^\zeta}{\zeta}}\, s_1 s_2 s_3\, H\left(\frac{1}{s_1},\frac{1}{s_2},\frac{1}{s_3}\right) ds_3\, ds_2\, ds_1 \tag{13}$$

Here the real constants c_1, c_2, c_3 are chosen so that all singularities of the integrand lie to the left of the lines $Re(s_1) = c_1$, $Re(s_2) = c_2$, $Re(s_3) = c_3$. This formula follows from the duality $H(u) = u\,F\left(\frac{1}{u}\right)$ between the Elzaki transform and the Laplace transform F. In practice, the inverse transform is obtained from the properties in Section 3.6 rather than by evaluating this integral.

3.5. Linearity Property of CTET

Suppose $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right) = H(u,v,w)$ and $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(g\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right) = G(u,v,w)$

and A, B and C are arbitrary constants, thus $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(Ah\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right) + Bg\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)$

$$= AE_\phi^\delta E_\varphi^\xi E_\psi^\zeta\, h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right) + BE_\phi^\delta E_\varphi^\xi E_\psi^\zeta\, g\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right) = AH(u,v,w) + BG(u,v,w)$$

Proof: From the definition of CTET, we have

$$E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(Ah\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)+Bg\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)$$

$$=uvw\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}-\frac{\varphi^\xi}{v\xi}-\frac{\psi^\zeta}{w\zeta}}\left(Ah\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)+Bg\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)d\phi\,d\varphi\,d\psi$$

$$=A\left(uvw\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}-\frac{\varphi^\xi}{v\xi}-\frac{\psi^\zeta}{w\zeta}}\,h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)d\phi\,d\varphi\,d\psi\right)+$$

$$B\left(uvw\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}-\frac{\varphi^\xi}{v\xi}-\frac{\psi^\zeta}{w\zeta}}\,h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)d\phi\,d\varphi\,d\psi\right)$$

$$=AE_\phi^\delta E_\varphi^\xi E_\psi^\zeta\,h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)+BE_\phi^\delta E_\varphi^\xi E_\psi^\zeta\,g\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$$

Hence, $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(Ah\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)+Bg\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)=AH(u,v,w)+BG(u,v,w)$

3.6. Fundamental Properties of CTET

(1) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta(C)=Cu^2v^2w^2$.

(2) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\left(\frac{\phi^\delta}{\delta}\right)^p,\left(\frac{\varphi^\xi}{\xi}\right)^q,\left(\frac{\psi^\zeta}{\zeta}\right)^r\right)=\Gamma(p+1)\Gamma(q+1)\Gamma(r+1)u^{p+2}v^{q+2}w^{r+2}$

$=\mathrm{p!\ q!\ r!}\ u^{p+2}v^{q+2}w^{r+2}$, $\quad p,q,r\in\mathbb{N}$

(3) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(e^{A\left(\frac{\phi^\delta}{\delta}\right)\pm B\left(\frac{\varphi^\xi}{\xi}\right)\pm C\left(\frac{\psi^\zeta}{\zeta}\right)}\right)=\left(\frac{u^2}{1\mp Au}\right)\left(\frac{v^2}{1\mp Bv}\right)\left(\frac{w^2}{1\mp Cw}\right)$

(4) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(sin\left(A\frac{\phi^\delta}{\delta}\right)sin\left(B\frac{\varphi^\xi}{\xi}\right)sin\left(C\frac{\psi^\zeta}{\zeta}\right)\right)=\left(\frac{Au^3}{1+A^2u^2}\right)\left(\frac{Bv^3}{1+B^2v^2}\right)\left(\frac{Cw^3}{1+C^2w^2}\right)$

(5) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(cos\left(A\frac{\phi^\delta}{\delta}\right)cos\left(B\frac{\varphi^\xi}{\xi}\right)cos\left(C\frac{\psi^\zeta}{\zeta}\right)\right)=\left(\frac{u^2}{1+A^2u^2}\right)\left(\frac{v^2}{1+B^2v^2}\right)\left(\frac{w^2}{1+C^2w^2}\right)$.

3.7. CTET of Partial Derivatives

Suppose $h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$ be a piece-wise continuous function on the domain $(0,\infty)\times(0,\infty)\times(0,\infty)$. For $\delta,\xi,\zeta\in(0,1]$, the CTET of the conformable fractional partial derivatives of orders δ,ξ and ζ are:

(1) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^\delta}{\partial\phi^\delta}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{u}H(u,v,w)-uH(0,v,w)$

(2) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^\xi}{\partial\varphi^\xi}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{v}H(u,v,w)-vH(u,0,w)$

(3) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^\zeta}{\partial\psi^\zeta}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{w}H(u,v,w)-wH(u,v,0)$

(4) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^{2\delta}}{\partial\phi^{2\delta}}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{u^2}H(u,v,w)-H(0,v,w)-u\frac{\partial}{\partial\phi}H(0,v,w)$

(5) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^{2\xi}}{\partial\varphi^{2\xi}}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{v^2}H(u,v,w)-H(u,0,w)-v\frac{\partial}{\partial\varphi}H(u,0,w)$

(6) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^{2\zeta}}{\partial\psi^{2\zeta}}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{w^2}H(u,v,w)-H(u,v,0)-w\frac{\partial}{\partial\psi}H(u,v,0)$

(7) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^{m\delta}}{\partial\phi^{m\delta}}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{u^m}H(u,v,w)-\sum_{i=0}^{m-1}u^{2-m+i}\,E_\varphi^\xi E_\psi^\zeta\left[\frac{\partial^{i\delta}}{\partial\phi^{i\delta}}h\left(0,\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right]$

(8) $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^{n\xi}}{\partial\varphi^{n\xi}}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{v^n}H(u,v,w)-\sum_{j=0}^{n-1}v^{2-n+j}\,E_\phi^\delta E_\psi^\zeta\left[\frac{\partial^{j\xi}}{\partial\varphi^{j\xi}}h\left(\frac{\phi^\delta}{\delta},0,\frac{\psi^\zeta}{\zeta}\right)\right]$

$$(9)\quad E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(\frac{\partial^{l\zeta}}{\partial\psi^{l\zeta}}\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)\right)=\frac{1}{w^l}H(u,v,w)-\sum_{k=0}^{l-1}w^{2-l+k}\;E_\phi^\delta E_\varphi^\xi\left[\frac{\partial^{k\zeta}}{\partial\psi^{k\zeta}}h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},0\right)\right]$$

Here $m,n,l\in\mathbb{N}$, and $\frac{\partial^{i\delta}}{\partial\phi^{i\delta}}$ denotes the i-fold sequential conformable derivative, with $\frac{\partial^0}{\partial\phi^0}h=h$ (similarly for φ and ψ)

Definition 4: The Heaviside unit step function in three variables is defined by

$$u\left(\frac{\phi^\delta}{\delta}-\alpha,\frac{\varphi^\xi}{\xi}-\beta,\frac{\psi^\zeta}{\zeta}-\gamma\right)=u\left(\frac{\phi^\delta}{\delta}-\alpha\right)\mathcal{U}\left(\frac{\varphi^\xi}{\xi}-\beta\right)u\left(\frac{\psi^\zeta}{\zeta}-\gamma\right)$$

$$=\begin{cases}1, & \frac{\phi^\delta}{\delta}>\alpha,\ \frac{\varphi^\xi}{\xi}>\beta,\ \frac{\psi^\zeta}{\zeta}>\gamma,\\ 0, & \text{otherwise.}\end{cases}$$

Theorem 3: If $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)=H(u,v,w)$, then

$$E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\;H\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\right)=e^{-\frac{\alpha^\delta}{u\delta}-\frac{\beta^\xi}{v\xi}-\frac{\gamma^\zeta}{w\zeta}}\;H(u,v,w)$$

Proof: By the definition of CTET,

$$E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\;H\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\right)$$

$$=uvw\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}-\frac{\varphi^\xi}{v\xi}-\frac{\psi^\zeta}{w\zeta}}\;h\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\;H\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\,d\phi\,d\varphi\,d\psi$$

Using Heaviside unit step function,

$$=uvw\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{\phi^\delta}{u\delta}-\frac{\varphi^\xi}{v\xi}-\frac{\psi^\zeta}{w\zeta}}\;h\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\;d\phi\,d\varphi\,d\psi$$

By substituting $\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta}=\frac{A^\delta}{\delta}$, $\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi}=\frac{B^\xi}{\xi}$, $\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}=\frac{C^\zeta}{\zeta}$,

$$=e^{-\frac{\alpha^\delta}{u\delta}-\frac{\beta^\xi}{v\xi}-\frac{\gamma^\zeta}{w\zeta}}uvw\int_0^\infty\int_0^\infty\int_0^\infty e^{-\frac{A^\delta}{u\delta}-\frac{B^\xi}{v\xi}-\frac{C^\zeta}{w\zeta}}\;h\left(\frac{A^\delta}{\delta},\frac{B^\xi}{\xi},\frac{C^\zeta}{\zeta}\right)\;A^{\delta-1}B^{\xi-1}C^{\zeta-1}\,dA\,dB\,dC$$

$$\therefore E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\;H\left(\frac{\phi^\delta}{\delta}-\frac{\alpha^\delta}{\delta},\frac{\varphi^\xi}{\xi}-\frac{\beta^\xi}{\xi},\frac{\psi^\zeta}{\zeta}-\frac{\gamma^\zeta}{\zeta}\right)\right)=e^{-\frac{\alpha^\delta}{u\delta}-\frac{\beta^\xi}{v\xi}-\frac{\gamma^\zeta}{w\zeta}}\;H(u,v,w)$$

Definition 5: Let $h_1\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$ and $h_2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$ be two real-valued piece-wise continuous function. The convolution of two functions h_1 and h_2 is symbolized by $(h_1\ ***\ h_2)\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)$ and defined by the triple integral

$$(h_1\ ***\ h_2)\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=\int_0^{\frac{\phi^\delta}{\delta}}\int_0^{\frac{\varphi^\xi}{\xi}}\int_0^{\frac{\psi^\zeta}{\zeta}}h_1\left(\frac{\phi^\delta}{\delta}-\alpha,\frac{\varphi^\xi}{\xi}-\beta,\frac{\psi^\zeta}{\zeta}-\gamma\right)\;h_2(\alpha,\beta,\gamma)d\alpha\,d\beta\,d\gamma$$

$$=\int_0^{\frac{\phi^\delta}{\delta}}\int_0^{\frac{\varphi^\xi}{\xi}}\int_0^{\frac{\psi^\zeta}{\zeta}}h_2\left(\frac{\phi^\delta}{\delta}-\alpha,\frac{\varphi^\xi}{\xi}-\beta,\frac{\psi^\zeta}{\zeta}-\gamma\right)\;h_1(\alpha,\beta,\gamma)d\alpha\,d\beta\,d\gamma$$

3.8. Convolution Theorem of CTET

Let $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h_1\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)=H_1(u,v,w)$ and $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h_2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)=H_2(u,v,w)$ exist, then the convolution theorem of CTET is $E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left(h_1\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)***h_2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right)=\frac{1}{uvw}\;H_1(u,v,w)H_2(u,v,w)$.

Proof: Using the definition of CTET

$$E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left((h_1 *** h_2)\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)=uvw\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-\frac{\phi^{\delta}}{u\delta}-\frac{\varphi^{\xi}}{v\xi}-\frac{\psi^{\zeta}}{w\zeta}}\,(h_1 *** h_2)\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)d\phi\,d\varphi$$

$$=uvw\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-\frac{\phi^{\delta}}{u\delta}-\frac{\varphi^{\xi}}{v\xi}-\frac{\psi^{\zeta}}{w\zeta}}\left(\int_0^{\frac{\phi^{\delta}}{\delta}}\int_0^{\frac{\varphi^{\xi}}{\xi}}\int_0^{\frac{\psi^{\zeta}}{\zeta}}h_1\left(\frac{\phi^{\delta}}{\delta}-\alpha,\frac{\varphi^{\xi}}{\xi}-\beta,\frac{\psi^{\zeta}}{\zeta}-\gamma\right)h_2(\alpha,\beta,\gamma)d\alpha\,d\beta\,d\gamma\right)d\phi\,d\varphi\,d\psi$$

By using Heaviside unit step function,

$$=uvw\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-\frac{\phi^{\delta}}{u\delta}-\frac{\varphi^{\xi}}{v\xi}-\frac{\psi^{\zeta}}{w\zeta}}\left(\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}h_1\left(\frac{\phi^{\delta}}{\delta}-\alpha,\frac{\varphi^{\xi}}{\xi}-\beta,\frac{\psi^{\zeta}}{\zeta}-\gamma\right)H\left(\frac{\phi^{\delta}}{\delta}-\alpha,\frac{\varphi^{\xi}}{\xi}-\beta,\frac{\psi^{\zeta}}{\zeta}-\gamma\right)h_2(\alpha,\beta,\gamma)\;d\alpha\,d\beta\,d\gamma\right)d\phi\,d\varphi\,d\psi$$

$$=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}h_2(\alpha,\beta,\gamma)\,d\alpha\,d\beta\,d\gamma\left(uvw\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}e^{-\frac{\phi^{\delta}}{u\delta}-\frac{\varphi^{\xi}}{v\xi}-\frac{\psi^{\zeta}}{w\zeta}}\,h_1\left(\frac{\phi^{\delta}}{\delta}-\alpha,\frac{\varphi^{\xi}}{\xi}-\beta,\frac{\psi^{\zeta}}{\zeta}-\gamma\right)H\left(\frac{\phi^{\delta}}{\delta}-\alpha,\frac{\varphi^{\xi}}{\xi}-\beta,\frac{\psi^{\zeta}}{\zeta}-\gamma\right)d\phi\,d\varphi\,d\psi\right)$$

Where $H\left(\frac{\phi^{\delta}}{\delta}-\alpha,\frac{\varphi^{\xi}}{\xi}-\beta,\frac{\psi^{\zeta}}{\zeta}-\gamma\right)=\begin{cases}1;\ \frac{\phi^{\delta}}{\delta}>\alpha,\frac{\varphi^{\xi}}{\xi}>\beta,\ \frac{\psi^{\zeta}}{\zeta}>\gamma\\ 0;\ \frac{\phi^{\delta}}{\delta}<\alpha,\ \frac{\varphi^{\xi}}{\xi}<\beta,\ \frac{\psi^{\zeta}}{\zeta}<\gamma\end{cases}$

Applying theorem (3) the above equation is converted to

$$E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left((h_1 *** h_2)\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)=\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}h_2(\alpha,\beta,\gamma)\,e^{-\frac{\alpha}{u}-\frac{\beta}{v}-\frac{\gamma}{w}}H_1(u,v,w)d\alpha\,d\beta\,d\gamma$$

$$E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left((h_1 *** h_2)\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)=\frac{1}{uvw}H_1(u,v,w)\left(uvw\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}h_2(\alpha,\beta,\gamma)\,e^{-\frac{\alpha}{u}-\frac{\beta}{v}-\frac{\gamma}{w}}d\alpha\,d\beta\,d\gamma\right)$$

$$E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left((h_1 *** h_2)\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)=\frac{1}{uvw}H_1(u,v,w)H_2(u,v,w)$$

3.9. Existence condition for CTET

Suppose $h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ be a function of exponential order on $(0,\infty)\times(0,\infty)\times(0,\infty)$. For $a>0,b>0,c>0$, then there is a constant $K>0$ such that $\forall\ \frac{\phi^{\delta}}{\delta}>X,\frac{\varphi^{\xi}}{\xi}>Y$, and $\frac{\psi^{\zeta}}{\zeta}>T$:

$$\left|h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right|\le Ke^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}$$

And we can write $h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=O\left(e^{a\left(\frac{\phi^{\delta}}{\delta}\right)+b\left(\frac{\varphi^{\xi}}{\xi}\right)+c\left(\frac{\psi^{\zeta}}{\zeta}\right)}\right)$ as $\frac{\phi^{\delta}}{\delta}\to\infty,\frac{\varphi^{\xi}}{\xi}\to\infty,\frac{\psi^{\zeta}}{\zeta}\to\infty$

Or, Equivalently,

$$\lim_{\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\to(\infty,\infty,\infty)}e^{-\alpha\frac{\phi^{\delta}}{\delta}-\beta\frac{\varphi^{\xi}}{\xi}-\gamma\frac{\psi^{\zeta}}{\zeta}}\left|h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right|\le K\lim_{\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\to(\infty,\infty,\infty)}e^{-(\alpha-a)\frac{\phi^{\delta}}{\delta}-(\beta-b)\frac{\varphi^{\xi}}{\xi}-(\gamma-c)\frac{\psi^{\zeta}}{\zeta}}=0,$$

$$\alpha>a,\ \beta>b,\ \gamma>c.$$

Hence, the CTET of h exists for $0<u<\frac{1}{a}$, $0<v<\frac{1}{b}$ and $0<w<\frac{1}{c}$.

3.10. Basic concept of the Adomian's method

Consider the fractional Equation [see 47]

$$h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=N\left(h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)+f\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right) \tag{14}$$

where N be a nonlinear operator and f is a well-known function. The Adomian method requires series solution

$$h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\sum_{n=0}^{\infty}h_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right) \tag{15}$$

where N can be decomposed as

$$N\left(h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)=\sum_{n=0}^{\infty}A_n\,(h) \tag{16}$$

Where $A_n\,(h)'s$ are Adomian polynomials with components $h_0,h_1,h_2,\cdots\ \cdots\ \cdots,h_n;n\geq 0$ defined by

$$A_n(h)=\frac{1}{n!}\left[\frac{d^n}{d\gamma^n}\left[F\left[\sum_{i=0}^{\infty}\gamma^i\,h_i\right]\right]\right]_{\gamma=0}\quad n=0,1,2,3,\cdots\cdots\cdots \tag{17}$$

Or, equivalently $A_0=F(h_0),\ A_1=h_1F'(h_0),\ A_2=h_2F'(h_0)+\frac{h_1^2}{2!}F''(h_0),\cdots\ \cdots\ \cdots\ \cdots$

From Equations (15) and (16), Equation (14) is converted to

$$\sum_{n=0}^{\infty}h_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=f+\sum_{n=0}^{\infty}A_n\,(h) \tag{18}$$

Equation (18) gives the following recurrence formula:

$$h_0=f_0 \tag{19}$$

$$h_1=A_0 \tag{20}$$

$$\vdots$$

$$h_{n+1}=A_n \tag{21}$$

$$\vdots$$

The exact solution of Equation (14) is now determined. But all the terms of the series cannot be determined and as a result, an approximate solution from the truncated series is defined as:

$$h=h_0+h_1+h_2+\cdots\cdots+h_{n-1},n>1 \tag{22}$$

3.11. Convergence Analysis

Theorem 4: [48] Let N be an operator from a Hilbert space $\mathcal{H}$ into $\mathcal{H}$, and let h be the exact solution of Equation (14). Then the series $\sum_{n=0}^{\infty}h_n$ in Equation (15) converges to h if there exists γ with $0\leq\gamma<1$ such that $\|h_{k+1}\|\leq\gamma\|h_k\|$ for all $k\in\mathbb{N}\cup\{0\}$.

Definition 6: [48], [49] For every $k\in\mathbb{N}\cup\{0\}$, we define

$$\alpha_k=\begin{cases}\frac{\|h_{k+1}\|}{\|h_k\|},if\ \|h_k\|\neq 0\\ 0\ ,if\ \|h_k\|=0\end{cases} \tag{23}$$

Therefore, the approximate series solution $\sum_{n=0}^{\infty}h_n$converges to h, when $0\leq\alpha_k<1$ for $k=0,1,2,3\ldots\ldots\ldots$.

3.12. Conformable Triple Elzaki Decomposition Method

To solve the considered problem in equation (2) by using suggested CTEDM, source term $g\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ is partitioned into two terms namely $g_1\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ and $g_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$. By CTET, first portion $g_1\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ consistently produces a simple algebraic expression and the second component $g_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ merged with nonlinear term to eliminate noise terms during the decomposition procedure. Subsequently, CTET and ADM are applied to handle the linear and nonlinear parts respectively.

To derive the analytical solutions of Equations (2) – (6), proposed method requires the following steps:

Step 1. Using the properties of CTET both sides of Equation (2)

$$\frac{1}{w^2}H(u,v,w)-H(u,v,0)-w\frac{\partial}{\partial\psi}H(u,v,0)+p\left(\frac{1}{w}H(u,v,w)-wH(u,v,0)\right)+qE_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left(N\left(h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)\right)=$$
$$r\left(\frac{1}{u^2}H(u,v,w)-H(0,v,)-u\frac{\partial}{\partial\phi}H(0,v,w)+\frac{1}{v^2}H(u,v,w)-H(u,0,w)-v\frac{\partial}{\partial\varphi}H(u,0,w)\right)+G_1(u,v,w)+ \quad (24)$$
$$E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left(g_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)$$

where $G_1(u,v,w)$ is the conformable triple Elzaki transformation of $g_1\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$.

Step 2. CDET is applied to the conditions (3), (4), (5), the obtained results are:

$$H(u,v,0)=A_1(u,v),\frac{\partial}{\partial\psi}H(u,v,0)=A_2(u,v) \quad (25)$$
$$H(0,v,w)=B_1(v,w),\frac{\partial}{\partial\phi}H(0,v,w)=B_2(v,w) \quad (26)$$
$$H(u,0,w)=B_3(u,w),\frac{\partial}{\partial\varphi}H(u,0,w)=B_4(u,w) \quad (27)$$

Step 3. Substituting Equations (25), (26) and (27) in Equation (24) and simplify, we get the following equation

$$H(u,v,w)=\frac{u^2v^2w^2}{u^2v^2+pu^2v^2w-rv^2w^2-ru^2w^2}\left[(1+pw)A_1(u,v)+wA_2(u,v)-r(B_1(v,w)+uB_2(v,w)+B_3(u,w)+vB_4(u,w)+\right.$$
$$\left.G_1(u,v,w)+E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left(g_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)-qN\left(h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)\right)\right] \quad (28)$$

Step 4. Taking inverse conformable triple Elzaki transform to the above Equation (28), then the equation becomes

$$h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2+pu^2v^2w-rv^2w^2-ru^2w^2}\left[(1+pw)A_1(u,v)+wA_2(u,v)-r(B_1(v,w)+uB_2(v,w)+\right.\right.$$
$$\left.\left.B_3(u,w)+vB_4(u,w)+G_1(u,v,w)+E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left(g_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)-qN\left(h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)\right)\right]\right] \quad (29)$$

Step 5. Assume that $h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=\sum_{n=0}^{\infty}h_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ (30)

is the series solution of the considered problem (Eq-2), where $N\left(h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)$ decomposed by Adomian polynomial defined in formula (17).

Therefore, $N\left(h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)=\sum_{n=0}^{\infty}A_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)$ (31)

A key strength of the proposed CTEDM is its capacity to effectively manage nonlinear terms while preserving

analytical tractability, leading to accurate solutions with reduced computational effort.

Step 6. Putting the Equations (30) and (31) in (29) gives

$$\sum_{n=0}^{\infty}h_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2+pu^2v^2w-rv^2w^2-ru^2w^2}\left[(1+pw)A_1(u,v)+wA_2(u,v)-r(B_1(v,w)+uB_2(v,w)+\right.\right.$$
$$\left.\left.B_3(u,w)+vB_4(u,w)+G_1(u,v,w)+E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left(g_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)-q\sum_{n=0}^{\infty}A_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)\right)\right] \quad (32)$$

Step 7. By equating both sides, the following results are obtained in accordance with equations (19) – (21).

$$h_0\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2+pu^2v^2w-rv^2w^2-ru^2w^2}\left[(1+pw)A_1(u,v)+wA_2(u,v)-r\left(B_1(v,w)+uB_2(v,w)+\right.\right.\right.$$
$$\left.\left.\left.B_3(u,w)+vB_4(u,w)\right)+G_1(u,v,w)\right]\right] \quad (33)$$

$$h_1\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2+pu^2v^2w-rv^2w^2-ru^2w^2}E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left(g_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)-qA_0\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)\right] \quad (34)$$

$$h_{n+1}\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=-qE_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2+pu^2v^2w-rv^2w^2-ru^2w^2}\left[E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left(A_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right)\right]\right],n\geq 1 \quad (35)$$

Step 8. Finally, the series solution of the given nonlinear telegraph equation (2) – (5) can be obtained using (22) as shown below

$$h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=h_0\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)+h_1\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)+\cdots\cdots\cdots+h_n\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right) \tag{36}$$

4. Illustrative Examples

In this section, the effectiveness and accuracy of the proposed CTEDM are examined through illustrative examples of nonlinear conformable fractional telegraph equations. The obtained solutions are compared with the exact solutions and those derived using the classical ADM. To evaluate the accuracy of the proposed method, the error measures that means Error, Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and Maximum Absolute Error are computed using the following formulas:

$$Error(x_i)=\left|h_{exact}(x_i)-h_{approx}(x_i)\right|$$
$$MAE=\frac{1}{n}\sum\left|h_{exact}(x_i)-h_{approx}(x_i)\right|$$
$$RMSE=\sqrt{\frac{1}{n}\sum(h_{exact}(x_i)-h_{approx}(x_i))^2}$$
$$Max\ Error=Max\left|h_{exact}(x_i)-h_{approx}(x_i)\right|$$

Here, h_{exact} denotes exact solution, h_{approx} denotes the approximate solution obtained by CTEDM or ADM, and n is the total grid points.

Example 1: Suppose conformable telegraph equation

$$\frac{\partial^{2\zeta}h}{\partial\psi^{2\zeta}}+2\frac{\partial^{\zeta}h}{\partial\psi^{\zeta}}=\frac{\partial^{2\delta}h}{\partial\phi^{2\delta}}+\frac{\partial^{2\xi}h}{\partial\varphi^{2\xi}}-h^2+e^{2(\phi^\delta/\delta)+2(\varphi^\xi/\xi)-4(\psi^\zeta/\zeta)}-2e^{(\phi^\delta/\delta)+(\varphi^\xi/\xi)-2(\psi^\zeta/\zeta)},\ \Omega=[0,2]\times[0,2] \tag{37}$$

with initial conditions $h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},0\right)=e^{(\phi^\delta/\delta)+(\varphi^\xi/\xi)},\frac{\partial^\zeta}{\partial\psi^\zeta}h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},0\right)=-2e^{(\phi^\delta/\delta)+(\varphi^\xi/\xi)}$ (38)

and boundary conditions $h\left(0,\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=e^{(\varphi^\xi/\xi)-2(\psi^\zeta/\zeta)},\frac{\partial^\delta}{\partial\phi^\delta}h\left(0,\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=e^{(\varphi^\xi/\xi)-2(\psi^\zeta/\zeta)}$ (39)

$$h\left(\frac{\phi^\delta}{\delta},0,\frac{\psi^\zeta}{\zeta}\right)=e^{(\phi^\delta/\delta)-2(\psi^\zeta/\zeta)},\frac{\partial^\xi}{\partial\varphi^\xi}h\left(\frac{\phi^\delta}{\delta},0,\frac{\psi^\zeta}{\zeta}\right)=e^{(\phi^\delta/\delta)-2(\psi^\zeta/\zeta)} \tag{40}$$

Applying CTET, equation (37) becomes

$$\frac{1}{w^2}H(u,v,w)-H(u,v,0)-w\frac{\partial}{\partial\psi}H(u,v,0)+2\left(\frac{1}{w}H(u,v,w)-wH(u,v,0)\right)=\frac{1}{u^2}H(u,v,w)-H(0,v,w)-u\frac{\partial}{\partial\phi}H(0,v,w)+\frac{1}{v^2}H(u,v,w)-H(u,0,w)-v\frac{\partial}{\partial\varphi}H(u,0,w)-\frac{2u^2v^2w^2}{(1-u)(1-v)(1+2w)}+E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left[e^{2(\phi^\delta/\delta)+2(\varphi^\xi/\xi)-4(\psi^\zeta/\zeta)}-h^2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right] \tag{41}$$

Applying CDET to the initial and boundary conditions, the following results are obtained

$$H(u,v,0)=\frac{u^2v^2}{(1-u)(1-v)},\frac{\partial}{\partial\psi}H(u,v,0)=\frac{-2u^2v^2}{(1-u)(1-v)} \tag{42}$$

$$H(0,v,w)=\frac{v^2w^2}{(1-v)(1+2w)},\frac{\partial}{\partial\phi}H(0,v,w)=\frac{v^2w^2}{(1-v)(1+2w)} \tag{43}$$

$$H(u,0,w)=\frac{u^2w^2}{(1-u)(1+2w)},\frac{\partial}{\partial\varphi}H(u,0,w)=\frac{u^2w^2}{(1-u)(1+2w)} \tag{44}$$

Equation (41) implies

$$H(u,v,w)=\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{\frac{u^2v^2}{(1-u)(1-v)}-\frac{v^2w^2+uv^2w^2}{(1-v)(1+2w)}-\frac{u^2w^2+u^2vw^2}{(1-u)(1+2w)}-\frac{2u^2v^2w^2}{(1-u)(1-v)(1+2w)}+E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left[e^{2(\phi^\delta/\delta)+2(\varphi^\xi/\xi)-4(\psi^\zeta/\zeta)}-h^2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right]\right\} \tag{45}$$

Simplifying,

$$H(u,v,w)=\frac{u^2v^2w^2}{(1-u)(1-v)(1+2w)}+\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left[e^{2\left(\frac{\phi^\delta}{\delta}\right)+2\left(\frac{\varphi^\xi}{\xi}\right)-4\left(\frac{\psi^\zeta}{\zeta}\right)}-h^2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right]\right\} \tag{46}$$

Taking inverse conformable fractional triple Elzaki transform, we obtain

$$h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=e^{\left(\frac{\phi^\delta}{\delta}\right)+\left(\frac{\varphi^\xi}{\xi}\right)-2\left(\frac{\psi^\zeta}{\zeta}\right)}+E_\phi^{\delta^{-1}}E_\varphi^{\xi^{-1}}E_\psi^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left[e^{2\left(\frac{\phi^\delta}{\delta}\right)+2\left(\frac{\varphi^\xi}{\xi}\right)-4\left(\frac{\psi^\zeta}{\zeta}\right)}-h^2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right]\right\}\right] \tag{47}$$

By Adomian's method and using equations (16), (19), (20), and (21), the parts of the solution are:

$$h_0\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=e^{\left(\frac{\phi^\delta}{\delta}\right)+\left(\frac{\varphi^\xi}{\xi}\right)-2\left(\frac{\psi^\zeta}{\zeta}\right)} \tag{48}$$

$$h_1\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=E_\phi^{\delta^{-1}}E_\varphi^{\xi^{-1}}E_\psi^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left[e^{2\left(\frac{\phi^\delta}{\delta}\right)+2\left(\frac{\varphi^\xi}{\xi}\right)-4\left(\frac{\psi^\zeta}{\zeta}\right)}-A_0\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right]\right\}\right]$$

$$=E_\phi^{\delta^{-1}}E_\varphi^{\xi^{-1}}E_\psi^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{E_\phi^\delta E_\varphi^\xi E_\psi^\zeta\left[e^{2\left(\frac{\phi^\delta}{\delta}\right)+2\left(\frac{\varphi^\xi}{\xi}\right)-4\left(\frac{\psi^\zeta}{\zeta}\right)}-e^{2\left(\frac{\phi^\delta}{\delta}\right)+2\left(\frac{\varphi^\xi}{\xi}\right)-4\left(\frac{\psi^\zeta}{\zeta}\right)}\right]\right\}\right]=0$$

$$h_{n+1}\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=-E_\phi^{\delta^{-1}}E_\varphi^{\xi^{-1}}E_\psi^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{A_n\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right\}\right],n\geq 1$$

Then,

$$h_2\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=-E_\phi^{\delta^{-1}}E_\varphi^{\xi^{-1}}E_\psi^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{A_1\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right\}\right]$$

$$=-E_\phi^{\delta^{-1}}E_\varphi^{\xi^{-1}}E_\psi^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+2u^2v^2w}\left\{2h_0h_1\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)\right\}\right]=0$$

$$h_3\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=h_4\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=h_5\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=\cdots h_k=0,k\geq 3$$

Therefore, by equation (22), solution of the considered example is

$$h\left(\frac{\phi^\delta}{\delta},\frac{\varphi^\xi}{\xi},\frac{\psi^\zeta}{\zeta}\right)=e^{\left(\frac{\phi^\delta}{\delta}\right)+\left(\frac{\varphi^\xi}{\xi}\right)-2\left(\frac{\psi^\zeta}{\zeta}\right)} \tag{49}$$

For $\delta=\xi=\zeta=1$, the two-dimensional conformable telegraph equation (37) reduces to the classical space–time telegraph equation, whose exact solution $h(\phi,\varphi,\psi)=e^{\phi+\varphi-2\psi}$ agrees with [33]. To construct a one-dimensional numerical comparison, we take $\delta=\xi=\zeta=1$ and fix $\phi=1$ and $\psi=1$, so that the solution reduces to $h(\varphi)=e^{\varphi-1}$, which is used to compute the values in Table 2.

Table 2. Numerical comparison of exact and approximate solutions using the proposed CTEDM and ADM.

φ	Exact Solution	CTEDM	ADM	Error (CTEDM)	Error (ADM)
0.5	0.6065	0.6065	0.6165	0	1.0×10^{-2}
1.0	1.0000	1.0000	1.0100	0	1.0×10^{-2}
1.5	1.6487	1.6487	1.6587	0	1.0×10^{-2}
2.0	2.7183	2.7183	2.7283	0	1.0×10^{-2}

Table 3. Error analysis for Example 1

Method	RMSE	MAE	Max Error
CTEDM	0.000000	0.000000	0.000000
ADM	1.00×10^{-2}	1.00×10^{-2}	1.00×10^{-2}

From Table 3, it is observed that the proposed CTEDM method produces results that agree exactly with the exact solution $h(x)=e^{\varphi-1}$, resulting in zero error. In contrast, the ADM method shows a uniform error of order 10^{-2} at all selected grid points. Hence, the RMSE, MAE, and Maximum Error values for ADM are identical. This confirms the superior accuracy of the proposed CTEDM method over ADM for Problem 1.

To test the convergence of the obtain solution, we compute α_k's for this problem using Definition 6 and Theorem 4 as follows:

$$\alpha_0 = \frac{\|h_1\|}{\|h_0\|} = \frac{\|0\|}{\left\| e^{\left(\frac{\phi^\delta}{\delta}\right)+\left(\frac{\varphi^\xi}{\xi}\right)-2\left(\frac{\psi^\zeta}{\zeta}\right)} \right\|} = 0 < 1.$$

$$\alpha_1 = \frac{\|h_2\|}{\|h_1\|} = 0 < 1, \ \ \alpha_2 = \frac{\|h_3\|}{\|h_2\|} = 0 < 1$$

Hence, for $\phi, \varphi \in [0,2]$ and $\psi \geq 0$, we obtain $\alpha_0 < 1$, $\alpha_1 < 1$, $\alpha_2 < 1$, and by induction $\alpha_k < 1$ for all $k \in \mathbb{N} \cup \{0\}$. Therefore, using Theorem 4, the solution of the given nonlinear conformable telegraph equation converges to the exact solution.

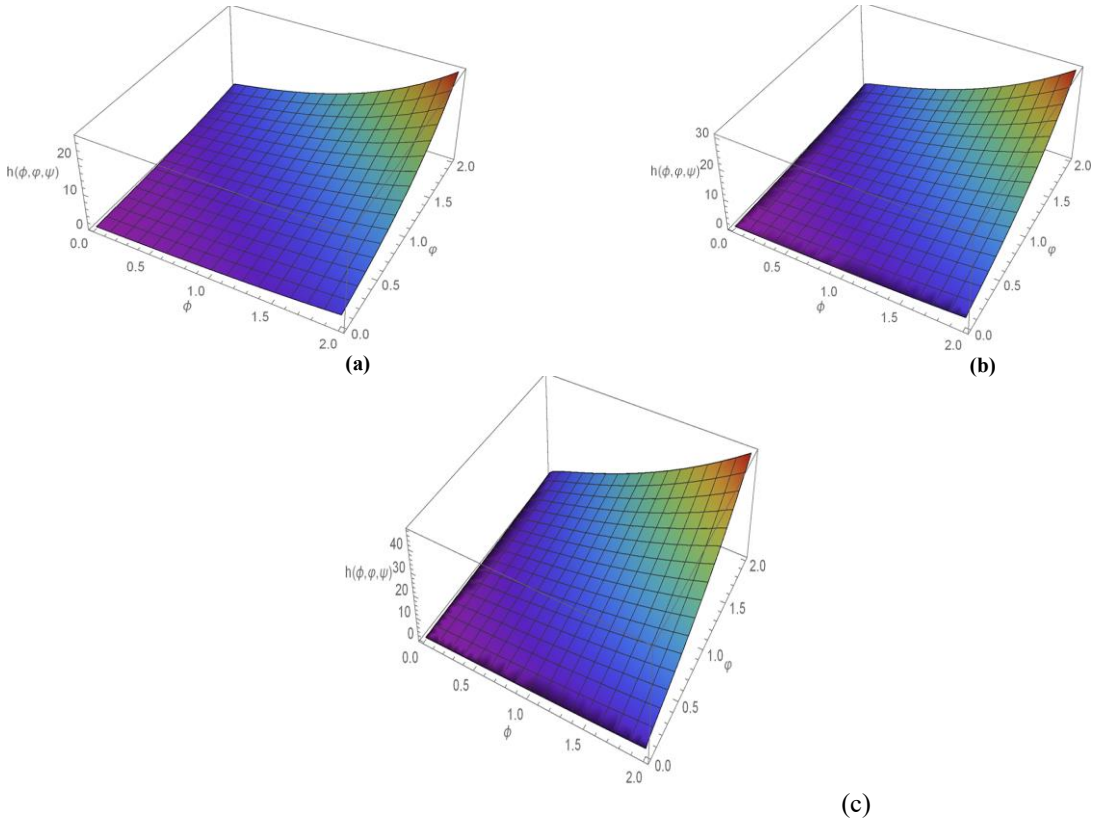


Figure 1. 3D surface plots of the solution $h(\phi, \varphi, \psi)$ of Example 1 as a function of ϕ and φ at $\psi = 0.4$, for (a) $\delta = \xi = \zeta = 0.9$, (b) $\delta = \xi = \zeta = 0.7$, (c) $\delta = \xi = \zeta = 0.5$.

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The three-dimensional (3D) graphical presentation of the exact solution for Example 1, corresponding to different fractional orders, is shown in Figure 1. Since the proposed CTEDM provides the exact analytical solution, so this graphical representation of exact solution is actually quite same as with CTEDM solution. Results in Table 2, Table 3 and Figure 1. show the capability of the proposed CTEDM to yield highly accurate solutions with low computational effort. Unlike ADM and other existing methods which need many iterations to improve their solutions while providing approximated results, the exact solution is instantly reached in a single iteration. The results verify that the proposed method is effective, stable, and exhibits rapid convergence.

Example 2: Consider the space-time fractional -order nonlinear telegraph equation with the external source term:

$$\frac{\partial^{2\zeta} h}{\partial \psi^{2\zeta}} + 5\frac{\partial^{\zeta} h}{\partial \psi^{\zeta}} + h^3 = \frac{\partial^{2\delta} h}{\partial \phi^{2\delta}} + \frac{\partial^{2\xi} h}{\partial \varphi^{2\xi}} - 2e^{-\left(\frac{\psi^\zeta}{\zeta}\right)} \sin\left(\frac{\phi^\delta}{\delta}\right)\cos\left(\frac{\varphi^\xi}{\xi}\right) + e^{-3(\psi^\zeta/\zeta)} \sin^3\left(\frac{\phi^\delta}{\delta}\right)\cos^3\left(\frac{\varphi^\xi}{\xi}\right) \quad (50)$$

with initial condition:

$$h\left(\frac{\phi^\delta}{\delta}, \frac{\varphi^\xi}{\xi}, 0\right) = \sin\left(\frac{\phi^\delta}{\delta}\right)\cos\left(\frac{\varphi^\xi}{\xi}\right), \frac{\partial^{\zeta}}{\partial \psi^{\zeta}} h\left(\frac{\phi^\delta}{\delta}, \frac{\varphi^\xi}{\xi}, 0\right) = -\sin\left(\frac{\phi^\delta}{\delta}\right)\cos\left(\frac{\varphi^\xi}{\xi}\right) \quad (51)$$

and boundary conditions:

$$h\left(0,\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=0,\frac{\partial^{\delta}}{\partial\phi^{\delta}}h\left(0,\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=e^{-(\psi^{\zeta}/\zeta)}\cos\left(\frac{\varphi^{\xi}}{\xi}\right) \tag{52}$$

$$h\left(\frac{\phi^{\delta}}{\delta},0,\frac{\psi^{\zeta}}{\zeta}\right)=e^{-(\psi^{\zeta}/\zeta)}\sin\left(\frac{\phi^{\delta}}{\delta}\right),\frac{\partial^{\xi}}{\partial\varphi^{\xi}}h\left(\frac{\phi^{\delta}}{\delta},0,\frac{\psi^{\zeta}}{\zeta}\right)=0 \tag{53}$$

Applying the properties of the CTET, equation (50) becomes

$$\frac{1}{w^2}H(u,v,w)-H(u,v,0)-w\frac{\partial}{\partial\psi}H(u,v,0)+5\left(\frac{1}{w}H(u,v,w)-wH(u,v,0)\right)=\frac{1}{u^2}H(u,v,w)-H(0,v,w)-u\frac{\partial}{\partial\phi}H(0,v,w)+\frac{1}{v^2}H(u,v,w)-H(u,0,w)-v\frac{\partial}{\partial\varphi}H(u,0,w)-\frac{2u^3v^2w^2}{(1+u^2)(1+v^2)(1+w)}+E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left[e^{-3(\psi^{\zeta}/\zeta)}\sin^3\left(\frac{\phi^{\delta}}{\delta}\right)\cos^3\left(\frac{\varphi^{\xi}}{\xi}\right)-h^3\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right] \tag{54}$$

Applying CDET to the initial and boundary conditions, we get the following results

$$H(u,v,0)=\frac{u^3v^2}{(1+u^2)(1+v^2)},\frac{\partial}{\partial\psi}H(u,v,0)=\frac{-u^3v^2}{(1+u^2)(1+v^2)} \tag{55}$$

$$H(0,v,w)=0,\frac{\partial}{\partial\phi}H(0,v,w)=\frac{v^2w^2}{(1+v^2)(1+w)} \tag{56}$$

$$H(u,0,w)=\frac{u^3w^2}{(1+u^2)(1+w)},\frac{\partial}{\partial\varphi}H(u,0,w)=0 \tag{57}$$

Substituting equations (55), (56), and (57) into equation (54), and simplifying, we have

$$H(u,v,w)=\frac{u^2v^2w^2}{u^2v^2+5u^2v^2w-v^2w^2-u^2w^2}\left\{\frac{u^3v^2+4u^3v^2w}{(1+u^2)(1+v^2)}-\frac{uv^2w^2}{(1+v^2)(1+w)}-\frac{u^3w^2}{(1+u^2)(1+w)}-\frac{2u^3v^2w^2}{(1+u^2)(1+v^2)(1+w)}+E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left[e^{-3(\psi^{\zeta}/\zeta)}\sin^3\left(\frac{\phi^{\delta}}{\delta}\right)\cos^3\left(\frac{\varphi^{\xi}}{\xi}\right)-h^3\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right]\right\} \tag{58}$$

Once again, by applying suitable mathematical manipulations to equation (58), it can be simplified to

$$H(u,v,w)=\frac{u^3v^2w^2}{(1+u^2)(1+v^2)(1+w)}+\frac{u^2v^2w^2}{u^2v^2+5u^2v^2w-v^2w^2-u^2w^2}\left\{E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left[e^{-3(\psi^{\zeta}/\zeta)}\sin^3\left(\frac{\phi^{\delta}}{\delta}\right)\cos^3\left(\frac{\varphi^{\xi}}{\xi}\right)-h^3\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right]\right\} \tag{59}$$

Taking inverse conformable fractional triple Elzaki transform, we obtain

$$h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=e^{-\left(\frac{\psi^{\zeta}}{\zeta}\right)}\sin\left(\frac{\phi^{\delta}}{\delta}\right)\cos\left(\frac{\varphi^{\xi}}{\xi}\right)+E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+5u^2v^2w}\left\{E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left[e^{-3(\psi^{\zeta}/\zeta)}\sin^3\left(\frac{\phi^{\delta}}{\delta}\right)\cos^3\left(\frac{\varphi^{\xi}}{\xi}\right)-h^3\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right]\right\}\right] \tag{60}$$

By Adomian's method and using (16), (19), (20), and (21), the parts of solution are:

$$h_0\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=e^{-\left(\frac{\psi^{\zeta}}{\zeta}\right)}\sin\left(\frac{\phi^{\delta}}{\delta}\right)\cos\left(\frac{\varphi^{\xi}}{\xi}\right) \tag{61}$$

$$h_1\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+5u^2v^2w}\left\{E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left[e^{-3(\psi^{\zeta}/\zeta)}\sin^3\left(\frac{\phi^{\delta}}{\delta}\right)\cos^3\left(\frac{\varphi^{\xi}}{\xi}\right)-A_0\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right]\right\}\right]$$

$$=E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+5u^2v^2w}\left\{E_{\phi}^{\delta}E_{\varphi}^{\xi}E_{\psi}^{\zeta}\left[e^{-3(\psi^{\zeta}/\zeta)}\sin^3\left(\frac{\phi^{\delta}}{\delta}\right)\cos^3\left(\frac{\varphi^{\xi}}{\xi}\right)-e^{-3(\psi^{\zeta}/\zeta)}\sin^3\left(\frac{\phi^{\delta}}{\delta}\right)\cos^3\left(\frac{\varphi^{\xi}}{\xi}\right)\right]\right\}\right]=0$$

$$h_{n+1}\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=-E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+5u^2v^2w}\left\{A_n\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right\}\right],n\geq 1 \tag{62}$$

Thus, $h_2\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=-E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+5u^2v^2w}\left\{A_1\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right\}\right]$

$$=-E_{\phi}^{\delta^{-1}}E_{\varphi}^{\xi^{-1}}E_{\psi}^{\zeta^{-1}}\left[\frac{u^2v^2w^2}{u^2v^2-v^2w^2-u^2w^2+5u^2v^2w}\left\{3h_0^2h_1\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)\right\}\right]=0$$

$h_3\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=h_4\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=h_5\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=0$ and so on.

Hence, by applying Equation (22), the solution for Example 2 can be expressed as

$$h\left(\frac{\phi^{\delta}}{\delta},\frac{\varphi^{\xi}}{\xi},\frac{\psi^{\zeta}}{\zeta}\right)=e^{-\left(\frac{\psi^{\zeta}}{\zeta}\right)}\sin\left(\frac{\phi^{\delta}}{\delta}\right)\cos\left(\frac{\varphi^{\xi}}{\xi}\right) \tag{63}$$

For $\delta=\xi=\zeta=1$, the two-dimensional nonlinear conformable telegraph equation (50) reduces to the classical space–time telegraph equation with exact solution $h(\phi,\varphi,\psi)=e^{-\psi}\sin(\phi)\cos(\varphi)$, which agrees with [37]. To construct a one-dimensional numerical comparison, we take $\delta=\xi=\zeta=1$ and fix $\varphi=1$ and $\psi=1$, so that the solution reduces to $h(\phi)=e^{-1}\cos(1)\sin(\phi)$, which is used to compute the values in Table 4.

Table 4. Numerical comparison of exact and approximate solutions using the proposed CTEDM and ADM.

ϕ	Exact Solution	CTEDM	ADM	Error (CTEDM)	Error (ADM)
0.5	0.0953	0.0953	0.1003	0	5.0×10^{-3}
1.0	0.1673	0.1673	0.1753	0	8.0×10^{-3}
1.5	0.1983	0.1983	0.2082	0	1.0×10^{-2}
2.0	0.1807	0.1807	0.1906	0	1.0×10^{-2}

Table 5. Error Analysis for Example 2

Method	RMSE	MAE	Max Error
CTEDM	0.000000	0.000000	0.000000
ADM	8.50×10^{-3}	8.25×10^{-3}	1.00×10^{-2}

The error values for the ADM method are not uniform in this case, leading to different values of RMSE and MAE. It is observed that the maximum error occurs at larger values of the independent variable, indicating gradual error growth. In contrast, the CTEDM method produces results that coincide exactly with the analytical solution $h(\phi) = e^{-1}cos(1)sin(\phi)$ resulting in zero error. This further confirms the superior accuracy and stability of the proposed method.

To test the convergence, we compute α_k's for this problem using Definition 6 and Theorem 4 as follows:

$$\alpha_0 = \frac{\|h_1\|}{\|h_0\|} = \frac{\|0\|}{\left\| e^{-\left(\frac{\psi^\zeta}{\zeta}\right)} \sin\left(\frac{\phi^\delta}{\delta}\right)\cos\left(\frac{\varphi^\xi}{\xi}\right)\right\|} = 0 < 1$$

$$\alpha_1 = \frac{\|h_2\|}{\|h_1\|} = 0 < 1,\ \alpha_2 = \frac{\|h_3\|}{\|h_2\|} = 0 < 1$$

Similarly, $\alpha_k < 1$. Therefore, using Theorem 4, the solution of the given example converges to the exact solution

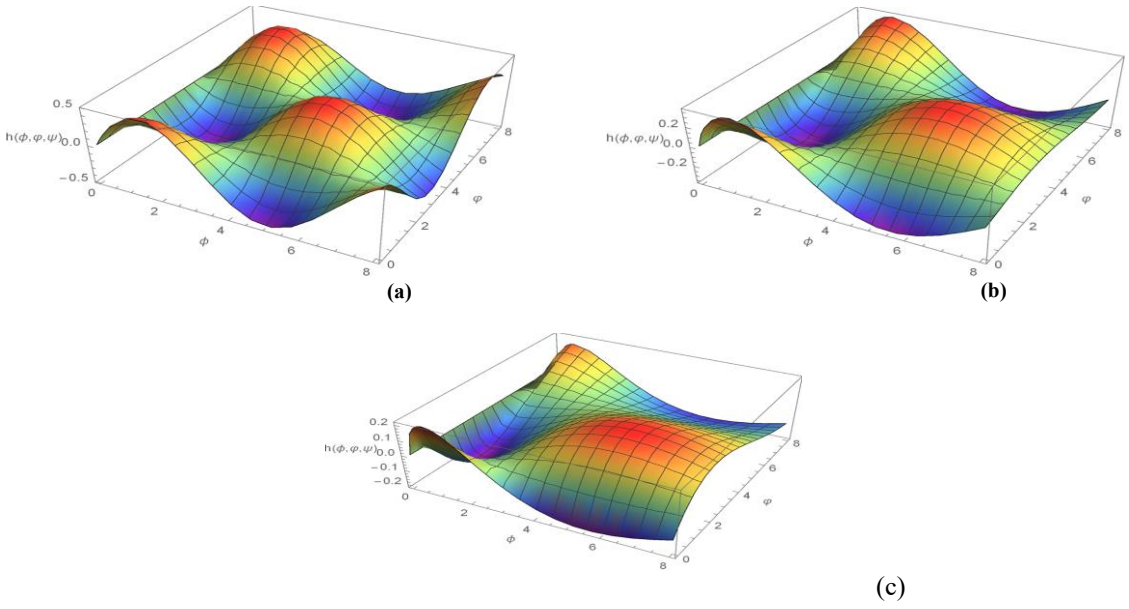


Figure 2. 3D surface plots of the solution $h(\phi, \varphi, \psi)$ of Example 2 as a function of ϕ and φ at $\psi = 0.6$, for (a) $\delta = \xi = \zeta = 0.9$, (b) $\delta = \xi = \zeta = 0.7$, (c) $\delta = \xi = \zeta = 0.5$.

The 3D graphical representation of the exact solution of Example 2 at different fractional orders and is depicted in Figure 2. Because the proposed CTEDM gets the same analytical solution, the plot of

exact solution is also used as a representation of CTEDM solution. As shown in Table 4, Table 5 and Figure 2., the proposed CTEDM results in highly accurate solutions with minor computation. The proposed method obtains the exact solution in one iteration, while existing methods like ADM deal with an approximation that usually needs multiple iterations to approximate on the right path. This illustrates the effectiveness, robustness and rapid convergence of the proposed method.

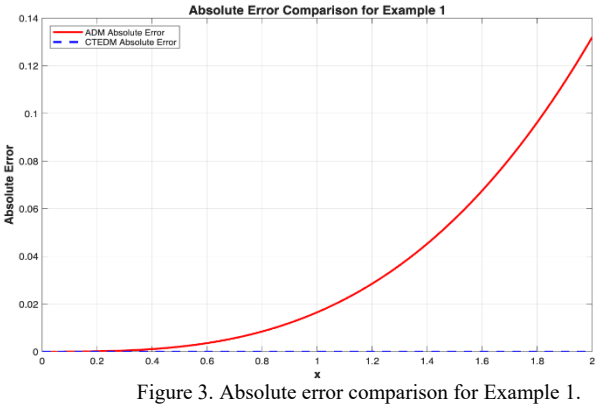


Figure 3. Absolute error comparison for Example 1.

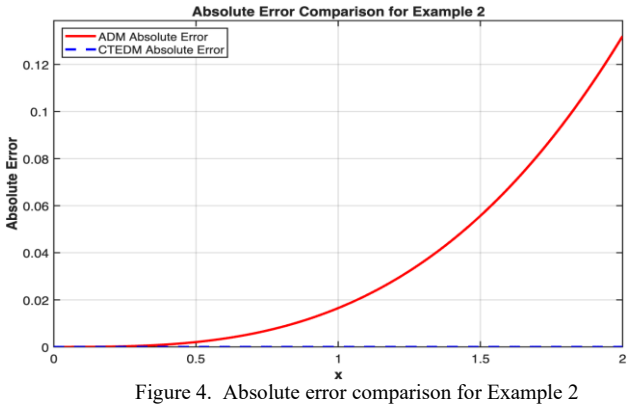


Figure 4. Absolute error comparison for Example 2

Figures 3 and 4 show the absolute errors of the ADM and the CTEDM for Examples 1 and 2, respectively. In both examples, the CTEDM error is zero throughout the domain, since the method yields the exact solution. In contrast, the ADM error increases with the independent variable, indicating that the truncation error of the ADM series accumulates over the domain.

5. Results and Discussion

Tables 2 and 4 compare the exact solution, the CTEDM solution and the ADM approximation at selected points for $\delta = \xi = \zeta = 1$, and Tables 3 and 5 summarize the corresponding error measures. In both examples, the CTEDM solution coincides with the exact solution at every grid point, so that the RMSE, MAE and maximum absolute error are all zero. In contrast, the ADM approximation shows errors of order 10^{-2}. In Example 1 these errors are uniform, so that the three error measures coincide, while in Example 2 they increase from 5.0×10^{-3} at $\phi = 0.5$ to 1.0×10^{-2} at $\phi = 2$, indicating that the truncation error of the ADM series accumulates over the domain. The zero error of the CTEDM is not a numerical coincidence: in both examples the CTEDM series terminates, with h_0 equal to the exact solution and $h_n = 0$ for all $n \geq 1$. This is due to two features of the method. First, the CTET converts the linear part of the equation, together with the initial and boundary conditions, into an algebraic equation, so that h_0 is obtained in closed form by a single inverse transform. Second, the part g_2 of the source term, which is combined with the nonlinear term, is exactly equal to the first Adomian polynomial, namely $A_0 = h_0^2$ in Example 1 and $A_0 = h_0^3$ in Example 2. Hence $g_2 - A_0 = 0$, which gives $h_1 = 0$, then $A_1 = 0$, and all higher components vanish. This cancellation of noise terms is a known feature of transform–decomposition methods [50], [51] and explains why a single iteration suffices in the present examples. The fractional orders enter the solutions (49) and (63) through $\frac{\phi^\delta}{\delta}$, $\frac{\varphi^\xi}{\xi}$ and $\frac{\psi^\zeta}{\zeta}$. For $0 < \delta < 1$ and $0 < \phi \leq 2$, one has $\frac{\phi^\delta}{\delta} > \phi$, so that decreasing the order stretches the variables; since $\frac{d}{d\phi}\left(\frac{\phi^\delta}{\delta}\right) = \phi^{\delta-1}$, this stretching is strongest near the origin. In Example 1, decreasing δ and ξ makes the solution grow faster in space, whereas decreasing ζ makes it decay faster in time (Figure 1). In Example 2, the amplitude factor $e^{-\psi^\zeta/\zeta}$ at $\psi = 0.6$ is approximately 0.50, 0.37 and 0.21 for $\zeta = 0.9$, 0.7 and 0.5, respectively (Figure 2), and the spatial oscillations become denser near the origin. Physically, a smaller fractional order therefore corresponds to stronger attenuation of the

voltage or current along the transmission line. For $\delta = \xi = \zeta = 1$, both solutions reduce to the classical solutions reported in [33], [37].

Overall, for the examples considered, the CTEDM yields the exact solution in a single iteration, without linearization or discretization, whereas the ADM requires a truncated series and produces errors of order 10^{-2}. These conclusions are based on two benchmark problems with known exact solutions; the limitations of the method are discussed in Section 6.

6. Limitations of the study

The present study has some limitations. First, the termination of the series after one iteration depends on a suitable splitting of the source term, $g = g_1 + g_2$, with $g_2 = A_0$; when such a splitting is not available, the method gives a truncated series whose accuracy depends on the number of terms retained. Second, the conformable derivative is a local operator, equivalent to a change of variables $\frac{\phi^\delta}{\delta}$, $\frac{\varphi^\xi}{\xi}$, $\frac{\psi^\zeta}{\zeta}$; hence it does not describe memory effects, and the results do not directly extend to nonlocal operators such as the Caputo derivative [17]. Third, the method requires functions of exponential order and closed-form inverse transforms, which may be difficult to obtain for complicated source terms or non-polynomial nonlinearities. Finally, the method has been tested only on two benchmark problems with known exact solutions, and error bounds for truncated series have not been derived. Problems with variable coefficients and without known exact solutions will be considered in future work.

7. Conclusion

In this study, the CTEDM was developed for solving two-dimensional nonlinear conformable fractional telegraph equations subject to initial and boundary conditions. Fundamental properties of the conformable triple Elzaki transform, including linearity, transforms of partial derivatives, the shift and convolution theorems and an existence condition, were established. The linear part of the equation was treated by the transform and the nonlinear part by Adomian polynomials. For the two examples considered, the method yielded the exact solutions in a single iteration, without linearization or discretization, and the solutions reduce to the classical ones when $\delta = \xi = \zeta = 1$. The three-dimensional plots show that smaller fractional orders lead to stronger attenuation of the signal. As discussed in Section 6, the single-iteration convergence relies on a suitable splitting of the source term, and the conformable derivative does not describe memory effects. Future work will extend the method to problems with variable coefficients, problems without known exact solutions, and higher-dimensional fractional models.

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Declarations

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Code availability. Not applicable. No custom research code is reported for the analytical derivations presented in this study.

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