American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)

ISSN (Print) 2313-4410, ISSN (Online) 2313-4402

http://asrjetsjournal.org/

Derivation of Klein – Gordon Equation for Frictional Medium

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Abstract

The expression for a wave function in a conducting medium together with relativistic energy momentum relation is used to derive Klien - Gordon equation for frictional medium. This equation reduces to the ordinary Klein- Gordon equation in free space. For good conducted lasing is possible. The amplification coefficient is proportional to conductivity.

Keywords: Friction; lasing; Klien-Gordon equation; conductivity.

1. Introduction

The Quantum friction plays an important role in determining the mechanical properties and the electrical properties of the matter .The most popular physical theory that is used to describe the physical properties of matter is quantum mechanics[1]. A prominent example is the theory of the quantized electromagnetic field applied to the case of two parallel moving plates separated by a small vacuum gap [2]. In this case the friction arises due to the spontaneous creation of particle pairs that propagate away into the plates or are dissipated there. A similar treatment can be applied to a body moving above a flat surface at constant speed [3,4]. Taking advantage of Lorentz invariance one achieves treatments consistent with special relativity [5, 6], as required for the archetypal situation that high-energy charges are stopped in a medium. According to that a recent research described such a formalism for a neutral, polarizable particle moving parallel to a flat interface [7].

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At a typical distance of at least a few nanometers ,then the interaction depends on a few macroscopic parameters (refractive index, conductivity, surface impedance . . .), these conditions make the friction a pure quantum-mechanical drag and closely relates it to the realm of Casimir phenomena [8]. A friction force appears when the speed of the particle relative to the surface exceeds the velocity of light in the medium this drag can thus be attributed to the Cherenkov effect. For a moving charge the Cherenkov drag is well known and is described easily with classical electromagnetic theory [9]. A neutral body requires a more refined treatment, as quantum fluctuations have to be treated accordingly. As in previous work , the useful of the fluctuation-dissipation theorem to do just that. Because of the growing interest in this field and some controversy surrounding it [10].

1.1. Derivation of Klein – Gordon Equation For Frictional Medium

According to Klein – Gordon equation :

The wave function of a free particle is given by:

$$\psi = e^{\frac{i}{\hbar}(Px - Et)} \tag{1}$$

Differentiating (1) with respect to t yields:

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \tag{2}$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = E^2 \psi \tag{3}$$

Differentiating (1) with respect to x gives

$$\frac{\hbar}{i} \nabla \psi = P\psi \tag{4}$$

$$-\hbar^2 \nabla^2 \psi = P^2 \psi \tag{5}$$

For frictional medium harmonic model [11] propose that

$$\psi = e^{\frac{i}{\hbar}(Px - Et + i\hbar \frac{\sigma}{\varepsilon} t)} \tag{6}$$

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}(E - i\hbar \frac{\sigma}{\varepsilon})\psi \tag{7}$$

Where the quantities σ and ε are respectively the conductivity and permittivity.

From equation (7)

$$i\hbar \left[\frac{\partial}{\partial t} + \frac{\sigma}{\varepsilon}\right] \psi = E\psi \tag{8}$$

Where the energy operator becomes

$$\hat{H} = i\hbar\frac{\partial}{\partial t} + i\hbar\frac{\sigma}{\varepsilon} \tag{9}$$

But from Klein – Gordon equation

$$E^2\psi = c^2P^2\psi + m_0^2c^4\psi \tag{10}$$

Thus the energy operator takes the form:

$$\hat{H}\psi = E\psi \tag{11}$$

Inserting equation(9) in equation (11) the energy eigen equation becomes

$$i\hbar\frac{\partial\psi}{\partial t} + i\hbar\frac{\sigma}{\varepsilon}\psi = E\psi \tag{12}$$

From equations (4), (10) and (12). one gets

$$\left(i\hbar\frac{\partial}{\partial t} + i\hbar\frac{\sigma}{\varepsilon}\right)^2\psi = c^2\left(\frac{\hbar}{i}\nabla\right)^2\psi + m_0^2c^4\psi \tag{13}$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} - 2\hbar^2\frac{\sigma}{\varepsilon}\frac{\partial\psi}{\partial t} - \hbar^2\frac{\sigma^2}{\varepsilon^2}\psi = -c^2\hbar^2\nabla^2\psi + m_0^2c^4\psi$$

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} - 2\hbar^2\frac{\mu\sigma}{\mu\varepsilon}\frac{\partial\psi}{\partial t} + c^2\hbar^2\nabla^2\psi = \hbar^2\frac{\sigma^2}{\varepsilon^2}\psi + m_0^2c^4\psi \tag{14}$$

For very poor conductor or insulator

$\sigma \to 0$

Thus one gets

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} + c^2\hbar^2\nabla^2\psi = m_0^2c^4\psi \tag{15}$$

Consider a photon moving inside a medium. It is equation can be solved by suggesting the solution

$$\psi = Ae^{i(\alpha x - \beta t)} \tag{16}$$

To get:

$$\frac{\partial^2\psi}{\partial t^2} = i^2\beta^2\psi = -\beta^2\psi$$

$$\nabla^2\psi = i^2\alpha^2\psi = -\alpha^2\psi \tag{17}$$

For a photon moving in free space or insulator one substitute (17) in (15) to get

$$\hbar^2\beta^2 - c^2\hbar^2\alpha^2 = m_0^2c^4 \tag{18}$$

But the wave equation for free particle is

$$\psi = Ae^{\frac{i}{\hbar}(Px-Et)} \tag{19}$$

A direct comparison of equation (19) with (16) gives:

$$P = \hbar\alpha \quad , \quad E = \hbar\beta \tag{20}$$

Inserting (20) in (18) gives

$$(E^2 - c^2P^2)\psi = (m_0^2c^4)\psi$$

$$E^2 = c^2P^2 + m_0^2c^4 \tag{21}$$

This is the ordinary energy – momentum relativistic relation. For a photon in a conductor however, substituting (18) in (14) yields

$$(\hbar^2\beta^2 + 2\hbar c^2\mu\sigma Ei - c^2\hbar^2\alpha^2)\psi = \left(\frac{\hbar^2\sigma^2}{\varepsilon^2} + m_0^2c^4\right)\psi \tag{22}$$

Using relation (20) in equation (22) yields

$$E^2 + 2\hbar c^2\mu\sigma Ei - c^2P^2 = \frac{\hbar^2\sigma^2}{\varepsilon^2} + m_0^2c^4 \tag{23}$$

Using relation (21), one can simplify (23) to get

$$2\hbar c^2\mu\sigma Ei = \frac{\hbar^2\sigma^2}{\varepsilon^2}$$

$$E = -\frac{\hbar\sigma}{2c^2\mu\varepsilon^2}i \tag{24}$$

According to special relativistic energy – momentum the energy given by

$$E = cP \tag{25}$$

Inserting equation (25) in equation (24) yields

$$P = -\frac{\hbar\sigma}{2c^3\mu\varepsilon^2}i = -\frac{\hbar\sigma}{2c\varepsilon}i \tag{24}$$

From equation (20)

$$\hbar\alpha = -\frac{\hbar\sigma}{2c\varepsilon}\mathrm{i} \tag{25}$$

$$\alpha = -\frac{\sigma}{2c\varepsilon}\mathrm{i} \tag{26}$$

Inserting equation (26) in (16) yields

$$\psi = Ae^{\frac{\sigma}{2c\varepsilon}x - i\beta t} \tag{27}$$

Thus the number of photons is given by

$$n = |\psi|^2 = \psi\overline{\psi} = A^2 e^{\frac{\sigma}{c\varepsilon}x} \tag{28}$$

This a gains means that lasing can take place.

1.2. Discussion

The Klein-Gordon equation for frictional medium shown in equation (14). For free space where the conductivity vanishes, the equation reduces to ordinary Klein–Gordon relativistic equation (see equation 14).However for conductor it predict that lasing can take place. This may be related to the fact that the conductivity is proportional to the number of free electrons. The increase of free electrons, increases collision which increases the number of excited atoms. This causes population inversion which leads to Lansing. This expression for lasing is similar to that obtained by some researchers.

1.3. Conclusion

Klein Gordon equation for frictional medium shows that lasing process possible in a conducting medium. It also shows that this equation reduces to Klein Gordon equation.

Acknowledgment

We would like to thank all those who assisted and encourage during the research period.

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