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Uncertainties and Optimization in Geotechnical Engineering

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Abstract

The needs of modern times for more economical, more efficient construction lead to a move away from the one-dimensional concept of "security" and are now turning towards the search for the "optimum" in search of the best solution through a set of secure options. Recognition of this need led to the creation of original mathematical programming techniques that focus on solving optimization problems. They include finding a set of variables that optimize the objective function and satisfy some predefined design constraints. Their capabilities have now been widely recognized, resulting in applications of genetic and other evolutionary algorithms, as well as neural networks extending not only to nonlinear problems but also to all aspects of geotechnical engineering. Studies of metaheuristic optimization algorithms have shown that they can provide appreciable results in geotechnical applications, such as the example of generating random fields of soil properties using the following L.A.S. method. It is estimated that their wider application to practical and theoretical geotechnical problems can bring beneficial results and become a particularly useful tool in the hands of civil engineers.

Keywords: uncertainties; optimization; metering algorithms; L.A.S.; autocorrelation; cross-correlation; reliability.

1. Introductory Concepts-Brief historical review

Every engineer knows that uncertainties in planning, designing and studying technical systems and projects are inevitable. It is therefore necessary for every engineer to apply the methods and concepts used to assess the importance of uncertainty in the study of technical systems.

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By this position, the principles of Probability and related fields of Statistics and modeling theory express the uncertainty and allow the analysis of its impact on the design of technical systems. In the sixteenth century, Gerolamo Cardano demonstrated the effectiveness of the definition of probabilities as the ratio of favorable to adverse effects. Apart from the basic work of Cardano, the doctrine of probabilities is dated to the correspondence of the Pierre de Fermat and Blaise Pascal (1654). Yakob Bernoulli (1713) and Abraham de Moivre (1718) treated the issue as a part of mathematics. The first two laws of the statistical error that are proposed, are derived fromPierre-Simon Laplace: the first law (1774) follows the Laplace distribution, while the second (1778) is the normal distribution (Gauss). Daniel Bernoulli in the 18th century, introduced the principle of the maximum product of the probabilities of a system of simultaneous errors. Adrien-Marie Legendre (1805) and Robert Adrain (1808) independently developed the least squares method. Andrey Markov introduced the concept of Markov Chains (1906), which played an important role in the theory of stochastic processes and its applications [1]. Finally, the modern Theory of Probabilities was developed by Andrey Kolmogorov (1931) [2].

2. Random variables and probability distributions

In Probability and Statistics, the probability distribution attributes a probability to every measurable subset of the different results of the random experiment, the survey, or the process of inductive statistics. In the applied probability, a probability distribution can be defined in a number of different ways. Often chosen for mathematical convenience:

- Probability density function
- Cumulative probability distribution function or probability distribution function

The most representative distribution is the Normal distribution, which is shown below:

3. Regular distribution data

3.1 Average value

The mean somehow represents all the possible values of a random variable.

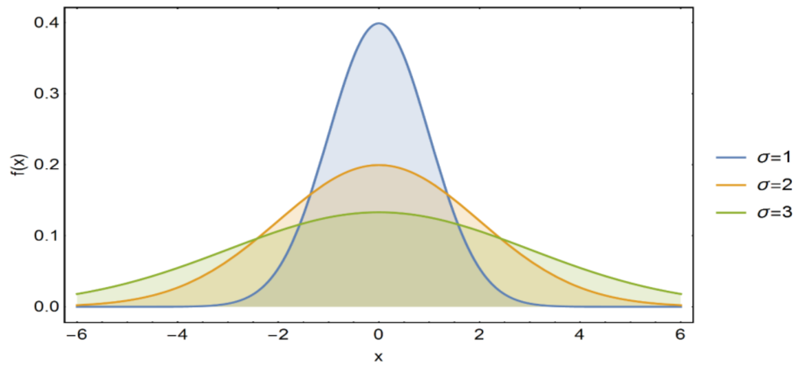


Figure 1: Probability density function of the normal distribution with mean value μ = 0 and standard dispersion σ = 1, 2, and 3.

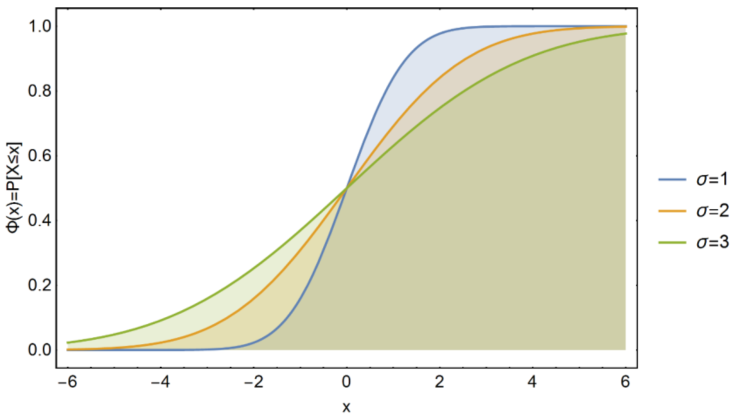


Figure 2: Cumulative probability density function of the normal distribution with mean value μ = 0 and standard dispersion σ = 1, 2, and 3.

3.2 Stochastic Variable Moments-Dispersion

The moments of a random variable provide additional useful information for the behavior of a random variable. Let X be a random variable.

The dispersion (or standard deviation) is a measure of whether the values of a random variable about its average are scattered. If the various possible values of the random variable are concentrated close to the mean value, the dispersion is small, and if sufficiently dispersed, the dispersion is large **[3].**

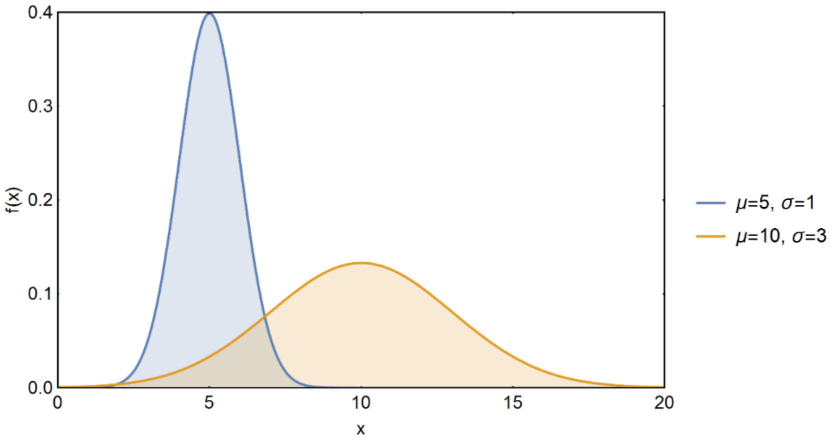


Figure 3: Effect of mean value and typical dispersion in the variance range of rates of two normal distributions.

3.3 Variability coefficient and correlation coefficient

The variability of the property values can be expressed by the coefficient of variation (υ)

$$\upsilon = \frac{\sigma_x}{\mu_x}$$

Frequently the model consists of more than two variables and it is possible that these variables interfere with each other. This influence is expressed using the term covariance. The covariance provides information about the type of relationship between the two variables and is calculated through the following equation:

$$Cov\,[X,\,Y] = E\,[XY] - E\,[X]\,E\,[Y\,] = E\,[XY\,] - \mu_X\,\mu_Y$$

The correlation coefficient calculates the linear dependence of X and Y. The correlation coefficient takes values from -1 to +1, when X and Y are absolutely linearly related **[4].**

$$\rho_{XY} = \frac{Cov[X, Y]}{\sigma_x \sigma_y}$$

Figure 4 shows the effect of the correlation coefficient ρ_{XY} in the density probability parity $f\,(x, y)$ for $\mu_x = 5,\ \mu_y = 5, \sigma_x = 1.5,\ \sigma_y = 2$.

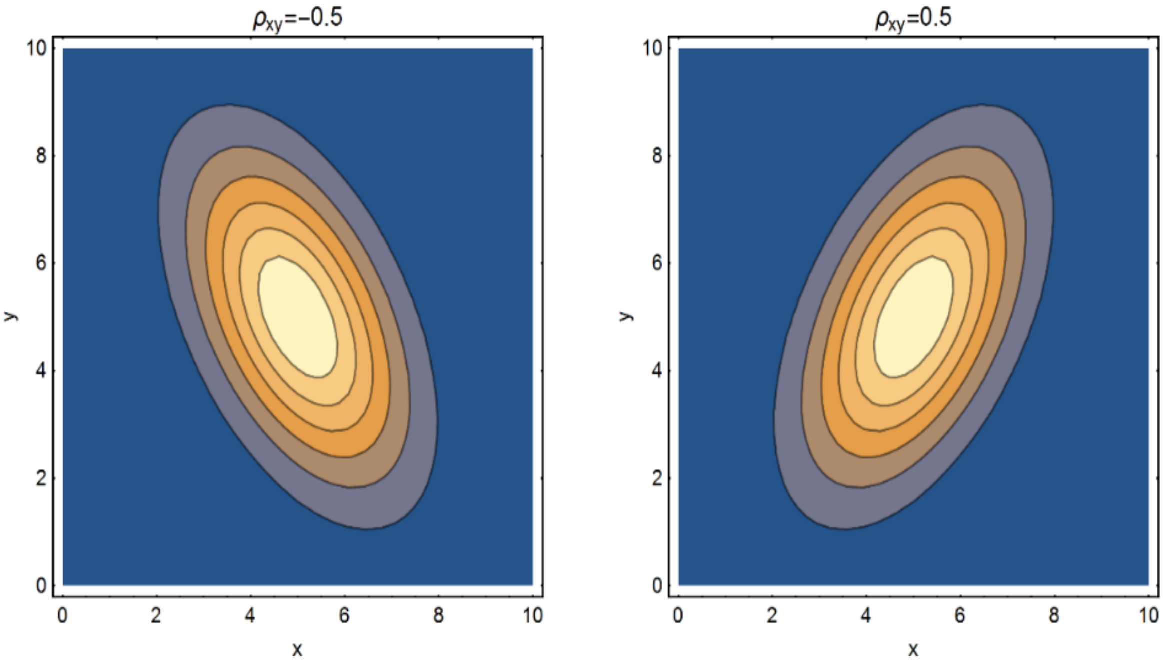


Figure 4: Effect of correlation coefficient ρ_{XY} in the density probability parity $f\,(x, y)$ for $\mu_x = 5,\ \mu_y = 5, \sigma_x = 1.5,\ \sigma_y = 2$: (a) ρ_{XY} = -0.5 and (b) 0.5

The above equations are used to describe the variance of data in a sample without taking into account the variance of the values in the space. In order to express the gradual change of parameters in space, it is necessary to perform spatial analysis.

This information is important considering the limitations of geotechnical design in estimating the value of a property at different points due to the uncertainties present in the soil.

3.4 Autocorrelation function and spatial correlation length

To express the statistical dependence of H values at different points x_1, x_2 which are located at a distance $\tau = |x_2 - x_1|$ (spatial length) the autocorrelation function is used

$$\rho\ (\tau) = \frac{Cov[\mathrm{H}(x_1), \mathrm{H}(x_2)]}{\sigma^2} = \frac{E[\left(\mathrm{H}(x_1) - \mu(x_1)\right)\left(\mathrm{H}(x_2) - \mu(x_2)\right)]}{\sigma^2}$$

To describe the spatial variability of an H property in one dimension, we use a characteristic length l_x from the spatial correlation in this dimension.

The length of spatial correlation l_x expresses the correlation between two random variables in the space with a correlation coefficient $\rho(\tau)$.

The above equation links the correlation coefficient between a random variable H following the normal distribution and the autocorrelation length:

$$\rho(\tau) = Exp[-\frac{|x_1 - x_2|}{l_x}] = Exp[-\frac{\tau}{l_x}]$$

Extending the random field of variable H in two dimensions leads to a self-correlation coefficient between two points (x_1, y_1) and (x_2, y_2) equal to

$$\rho = Exp[-\frac{|x_1 - x_2|}{l_x} - \frac{|y_1 - y_2|}{l_y}]$$

where l_x, l_y the characteristic lengths of autocorrelation in x and y directions, respectively **[5].**

3.5 Statistical parameters of mechanical characteristics of the soil

Following, the indicative fluctuation ranges of the mean μ, as well as the coefficient of variation Cov of the effective internal friction angle φ, the active cohesion c, as well as the unit weight γ (specific gravity), soil masses, are presented as found in the literature. More specifically, the coefficient of variation for the active internal friction angle registered between 2% and 15%, as shown in the table below. There is not enough data on the variation in unit weight. Smith and his colleagues (1995) [18], Hicks and his colleagues (2002) [19] & Griffiths and his colleagues (2002) [13], considered a deterministic weight unit variable of 20 KN/m^3.

Table 1: Average values μ and coefficient of variation Cov for the active angle of internal friction

Researcher	Year	μ	Cov
Harr [6]	1987		2% - 13%
Kalhawy [7]	1992		2% - 13%
Phoon and his colleagues [8]	1995	20 - 40 (deg)	5% - 15%
Lacasse and his colleagues [9]	1997		2% - 5%
Suchomel [10]	2010	21 (deg)	8%
Phoon and his colleagues [8]	1999	21-40 (deg)	5% - 15%
Duncan [11]	2000		2% - 13%
Jeremic and his colleagues [12]	2007		2% - 5%
Griffiths and his colleagues [13]	2002	35 (deg)	5% - 50%
El Ramley and his colleagues [14]	2002	35 (deg)	5.60%
Schweiger [15]	2005	35 (deg)	0

Table 2: Average values of μ and coefficient of variation Cov for active cohesion.

Researcher	Year	μ	Cov
Griffiths and his colleagues [13]	2002	$24KN/m^2$	30%
Suchomel [10]	2010	$10KN/\ m^2$	21%
Harr [6]	1987		20%
Cherubini [16]	1997		20%-30%
Li and his colleagues [17]	1987		40%

Table 3: Average values μ and coefficient of variation Cov for unit weight.

Researcher	Year	$\mu(kN/m^3)$	Cov
Harr [6]	1987		1%-10%
Phoon and his colleagues [8]	1995	13-20	<10%
Smith and his colleagues [18]	2004	20	0%
Duncan [11]	2000	14-20	<10%
Wang and his colleagues [20]	2010	20	6%
Hicks and his colleagues [19]	2002	20	0%
Griffiths and his colleagues [13]	2002	20	0%
Schweiger [15]	2005	20	0%

Finally, R. Rackwitz (2000) [21], proposes the following standard deviation values:

Table 4: Standard deviation of shear resistance parameters

Endurance parameters	Standard deviation
Specific weight (kN/m^3)	1
Angle of internal friction	4-8
Consistency (kPa)	6-15
Shear measure (MPa)	7-28

Vorechovsky (2007) [22], emphasizes that a change in mean value, standard deviation and correlations has an effect on both the autocorrelation coefficient and the correlation coefficient.

Of particular interest are the values of the cross-correlation coefficient ρ between the shear strength parameters of the soil (cohesion, internal friction angle) and the specific weight γ of the soil material as shown in the following tables:

Table 5: Cross-correlation of c and φ

Researcher	Year	$\rho_{c\varphi}$	Test
Forrest and Orr [23]	2010	-0,47	
Harr [6]	1987	0,25	CU
Hara and his colleagues [24]	2011	-0,1	CD
" "		-0,81	CD
" "		-0,87	CD
" "		-0,572	
" "		-0,554	
" "		-0,49	
" "		-0,359	
" "		-0,557	
Lumb [25]	1970	-0,7	
" "		-0,37	
Matsuo and Kuroda [26]	1974	-0,412	Direct shear test
		0,316	" "
		0,369	" "
		-0,474	" "
		-0,748	" "
Wolff [27]	1985	-0,47	CD

Table 6: Cross-correlation of c and φ

Researcher	Year	$\rho_{c\gamma}$	Test
Babu and Srirastava [28]	2006	0,25	
" "		0.5	
" "		0,75	
Chowdhurry and Xu [29]	1995	0,4	
Low and Tang [30]	1997	0.5	
Matsuo and Kuroda [26]	1974	0,44	Direct shear test

Table 7: Cross - correlation of φ and γ (Xing Zheng Wu 2011)

Researcher	Year	$\rho_{\gamma\varphi}$
Babu and Srirastava [28]	2007	0,25
" "		0.5
" "		0,75
Chowdhurry and Xu [29]	1992	0,7
Low and Tang [30]	1997	0,59
Matsuo and Kuroda [26]	1974	0,713
" "		0,656
" "		0,926
" "		0,859
" "		-0,943

The value of the correlation coefficient between the specific gravity and the internal friction angle according to Rakwitz (2000) is slightly positive between 0 and 0.5, whereas the correlation coefficient between the cohesion and the internal friction angle is always negative at -0.5.

If all the necessary spatial and point statistical parameters are available, probability analysis can be completed using various methods found in the literature [31].

4. Uncertainties and optimization in Geotechnical engineering

The needs of modern times for more economical, more efficient construction lead to a move away from the one-dimensional concept of "security" and are now turning towards the search for the "optimum". In search, that is, of the best solution through a set of secure options.

Optimization defines the process of finding the best possible solution to a problem under specific conditions and constraints. The goal of an optimization process is either to minimize the cost or process that needs to be paid for an effort, or to maximize the benefit we can get from one process. The required effort or the desired benefit of a project can be approximated by the objective function of the problem. The optimization problem then can be defined as finding the maximum or minimum of this particular function that describes the problem with some specific decision variables. Of course, the geotechnical engineering sector could not remain unaffected by this tendency and gradually the conventional empirical methods used by geotechnical engineers for most practical applications are replaced by new innovative methodologies.

Real applications of civil engineering often involve uncertainties due either to aleatory uncertainty or to the epistemic uncertainty of the problems encountered [32]. These uncertainties often play a decisive role, especially in the performance-based design of seismic engineering, such as geotechnical seismic applications (e.g. sloping movements), that are highly stochastic problems.

Over the past few years, however, with the very important development of stochastic analysis but with the increase of computational capabilities, the probabilistic methods are continuously gaining ground. Recognition of this need led to the creation of original mathematical programming techniques that focus on solving optimization problems. They include finding a set of variables that optimize the objective function and satisfy some predefined design constraints. Over the last thirty years, more and more researchers focused their interest on the use of such methods in applications such as the calculation of soil stress, geo-construction durability, slope stability etc. Although the difficulties faced initially were overwhelming due to the limited computing capabilities, the superiority of optimization methods in solving nonlinear problems, either of equal or unequal nature, was obvious.

Potchman and Kolesnichenco (1972) [33], they used dynamic programming for capacity problems, while Krugman and Krizek (1973), [34], as well as Chen (1975) [35] solved slope stability issues with two to three variables. Optimisation and other evolutionary algorithms are useful methods for analysis and design. Plus, their capabilities have widely recognized resulting in the applications of evolutionary algorithms to extend not only to nonlinear problems but also in all areas of Geotechnical Engineering [36]. The different types of uncertainties and errors in geotechnical engineering are shown in Figure 5.

At international conferences as well as in reputable journals, assumptions around the safety factor have been enriched and expanded by the results of a stochastic analysis. The development of these methods contributes to more economic and security-friendly approaches.

Another interesting point is the description of the estimation of uncertainty which is introduced with the arrangement of the perception of the geological soil model.

After an on-site investigation, the engineer's interest lies in assessing the values (parameters of mechanical strength) in soils from which samples were not taken. One could choose a process such as interference with the purpose of identifying the properties of soil parameters in non-sampled areas in the vicinity. These methods are based on the assumption of spatial variability. In contrast to interference there are the approaches to geostational simulations, schematically described by Phoon & Kulhawy (1999) [8] and then by Honjo (2002) [37] and Baecker & Cristian (2003) [38] as follows :

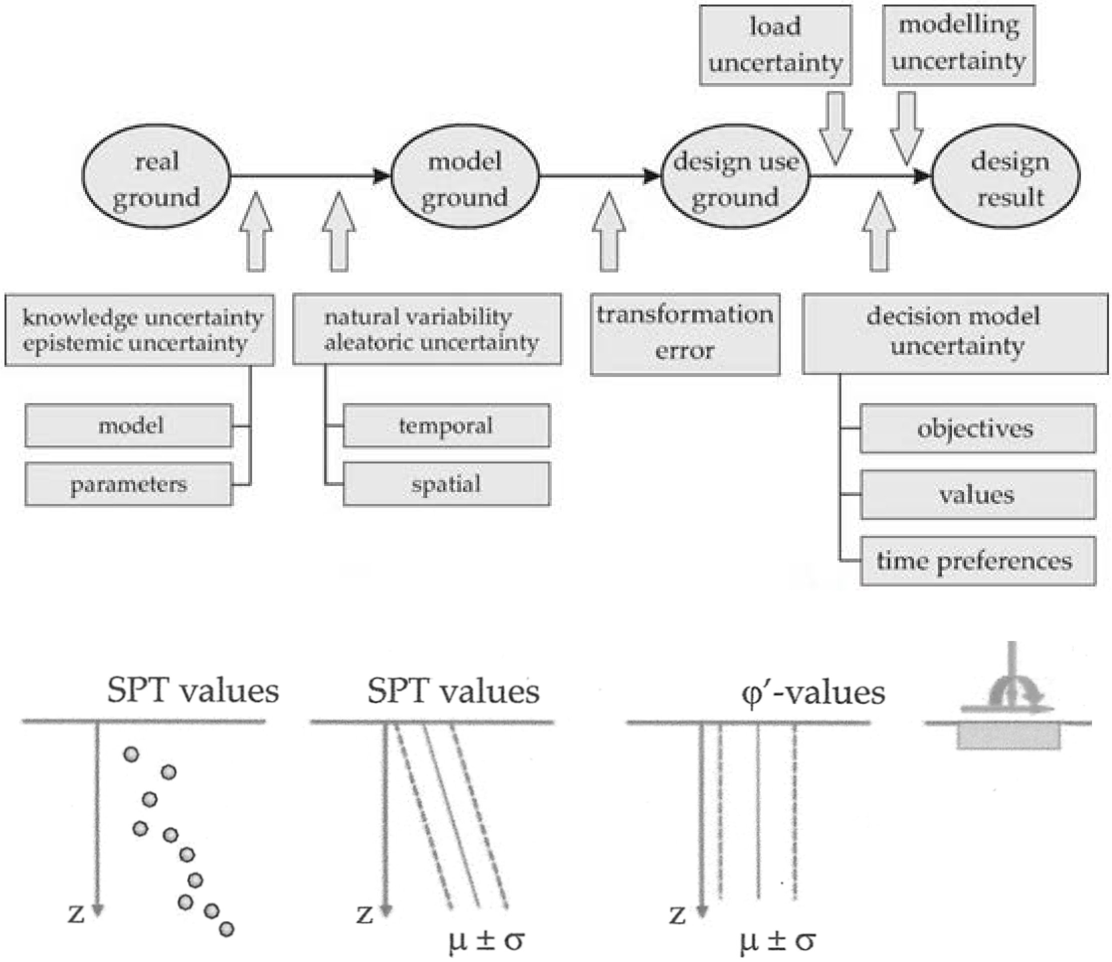


Figure 5: Uncertainties in the reliability of geotechnical design by Baecker & Cristian, Honjo, Phoon, Phoon & Kulhawy.

5. Analysis of reliability in Geotechnical Engineering

The geotechnical design is one of the subjects of study for civil engineers, including a great deal of uncertainty because of the natural heterogeneity of soil materials and the limited survey work [39].

The safety factors used in conventional geotechnical engineering are based on empirical methods, while a security factor value is often used without taking into account the uncertainty contained in the calculation. Because of current regulations and tradition, using the same coefficient for some applications has taken hold, even though the circumstances have a very wide range of degrees of uncertainty.

The theoretical background and analysis of reliability of constructions have evolved significantly over the last 30 years, this being reflected in the growing number of publications related to this topic. These developments, coupled with more accurate quantification of uncertainties associated with load and strength of construction, have stimulated interest in the probabilistic view of structures. The reliability of a structure or the chance of failure is a key factor in the design process, after considering the possibility to successfully fulfill the construction design requirements. Reliability analysis leads to additional security measures that the design engineer should take into account, due to the above-mentioned uncertainties [40].

Reliability analyses provide a means of assessing the effects of uncertainties, but also a way of distinguishing between conditions where uncertainties are typically large or small. Although the theory of reliability analysis has a potentially exceptionally promising, it has not been used and exploited in practice as it should. This is mainly due to two reasons. Firstly, it includes terms and concepts with which most geotechnical engineers are not familiar and secondly, there is a general belief that the reliability analysis requires more effort and a larger amount of data than what is available in usual practical geotechnical problems [41].

Many researchers [42] have described excellent examples of reliability analysis as is used in geotechnical applications, simultaneously quoting clear and detailed explanations of the underlying concepts. At the same time, Duncan (2000) attempted to show that the theories of reliability can be applied in simple ways so as not to require more effort and data volume than conventional methods.

It is widely recognized that the properties of soil parameters change even in homogeneous soil layers. The randomness and the spatial variability of soil parameters, correlation functions (between cohesion and internal friction angle) and auto-correlation in the horizontal and vertical direction, received special attention from many researchers who studied their effect in various geotechnical systems. For this reason, a solution based on the Local Average Subdivision (LAS) [43] by creating random fields of soil properties is presented.

6. Create random sloping soil properties with the LAS method

The Local Average Subdivision methodology (LAS), is used in this paper to create random fields of soil properties with default values for the mean μ </bx>, typical dispersion σ coefficients of ρ_{ij} correlation between properties of i and j, and the autocorrelation l_x lengths l_y in the horizontal and vertical direction, respectively. The numerical code implementation in the LAS methodology was created by modifying an existing code by Fenton and Griffiths (2008) [4] for geotechnical analysis systems (e.g. surface failure of foundations) by use of the finite element method and with the Monte Carlo simulation type.

In the methodology presented in this thesis, the production of random property fields is automated with the help of the Mathematica program. **[44].** Specifically, for each analysis, a random four-digit number that serves as a genetic number (seed number) to enable LAS methodology is initially created by the Mathematica program. The necessary data for the LAS analysis are:

1. The bands of material, are in number equal to nx in the horizontal direction and ny in the vertical direction, while each zone has dx dimensions in the horizontal direction and dy in the vertical direction.
2. The average value μ , formal dispersion σ , and the kind of distribution (normal, logarithmic,

uniform) for each property of the soil, such as cohesion c , shear strength angle φ, angle of expansion ψ, elastic modulus e, density ρ, and Poisson's ratio ν.

3. The register of correlation coefficients ρ_{ij} between the above properties (6x6).
4. Autocorrelation lengths l_x both l_y in the horizontal and vertical directions, respectively.
5. The random four-digit genetic number.

The LAS program executes as a subroutine from Mathematica and generates random values of soil properties that meet the above specified properties [45].The results of the analysis with the fields of random properties are graphically represented automatically in the Mathematica environment.

7. Confirmation of the LAS methodology

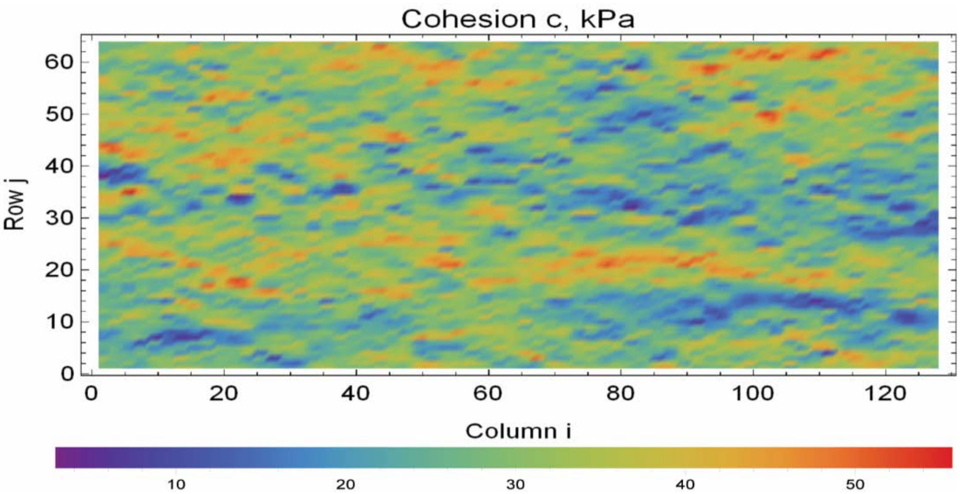
Figure 6 shows an example of a random field of coherence c and shear strength angle ϕ of the ground, with a grid of 128 x 64, in which the lengths of each zone are dx = dy = 1 m. Correspondingly, Figure 7 presents random density ranges ρ and Young's modulus of elasticity E. Average values and standard dispersions of soil properties are given in Table 1, and the correlation coefficients are given in Table 2. The autocorrelation lengths are l_x = 20 m in the horizontal direction and l_y = 2 m in the vertical direction. Along with the precise (targeted) average values and standard dispersions (table 1) and correlation coefficients (table 2), the corresponding values obtained numerically through simulation are also given. The comparison of targeted and attained rates is very satisfactory for the average values and the correlation coefficients, and quite satisfactory for the typical dispersions. Consequently, the LAS methodology achieves the goal of generating random property domains with desirable quantitative characteristics [46].

Table 8: Soil properties in the analyses of Figure 6

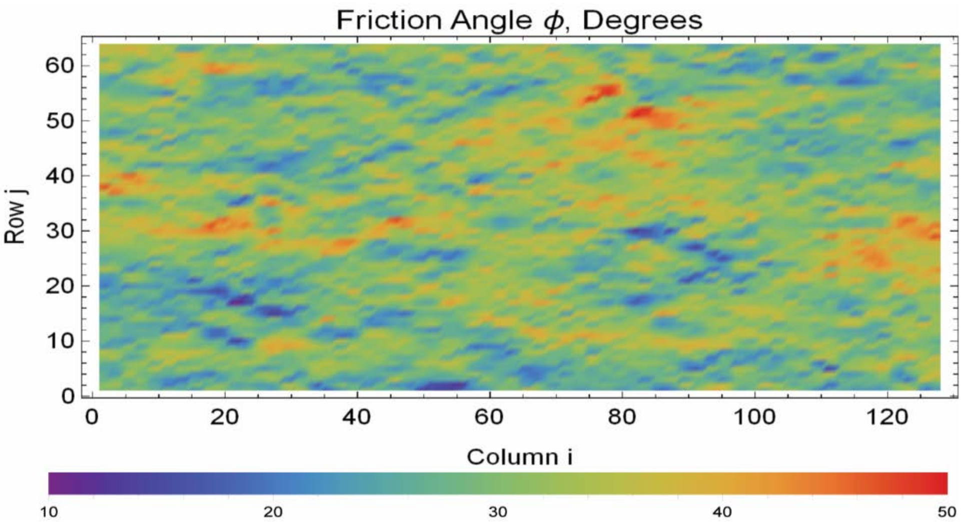
	Exact mean value, μ	**Attained mean value, μ**	**Exact dispersion, p**	**Attained dispersion, p**
Cohesion c (kPa)	30	30.0746	9	7.579
Shear strength angle φ°	30°	30.0096°	6°	5.111
Angle of expansion ψ°	0°	0	0°	0
Density, ρ (kg / m^3)	2000	2.0019	200	172
Young E modulus (kPa)	60000	59687	12000	10628
Poisson v modulus	0.3	0.3	0	0

Table 9: Correlation coefficients ρ_{ij}

	Exact value	Attained value
$\rho_{c\phi}$	-0.5	-0.5075
$\rho_{c\rho}$	0.5	0.5228
$\rho_{\phi\rho}$	0.5	0.4920
ρ_{cE}	0.2	0.1946
$\rho_{\phi E}$	0.2	0.2097

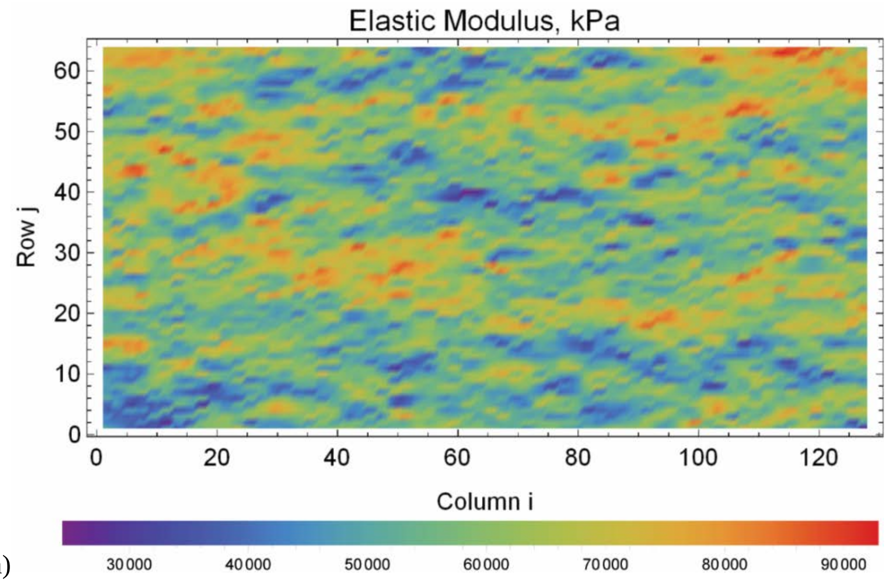


(a)

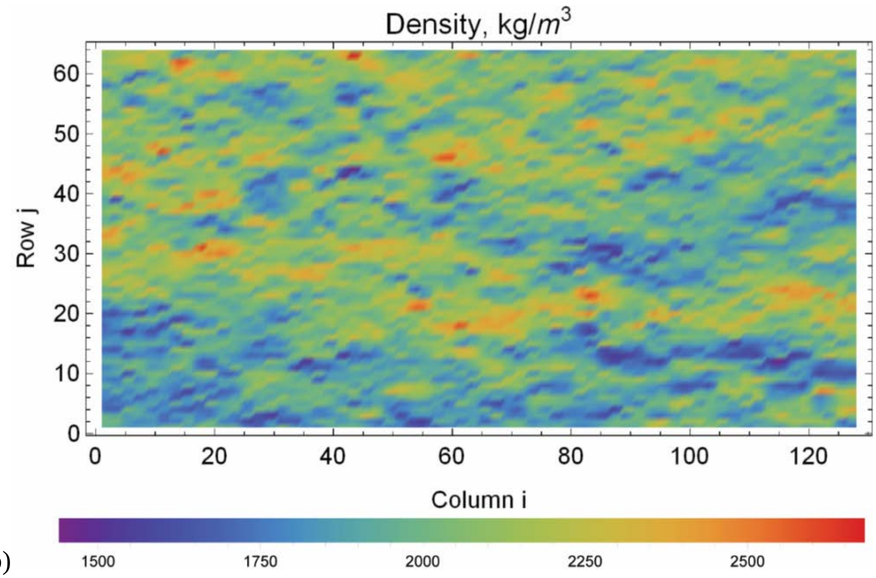


(b)

Figure 6: Create a field of random values (a) of cohesion c and (b) the shear strength angle ϕ with targeted property values given in Tables 8 and 9.



(a)



(b)

Figure 7: Create a range of random values (a) of the Young E elastic modulus and (b) the density ρ with the targeted property values given in Tables 8 and 9.

As expected, the values of cohesion c, shear strength ϕ and the values of Young E elastic modulus and the density ρ of Figures 6 and 7 demonstrate the widening of random field values by increasing the standard spreading ratio.

8. Constraints

It should be emphasized that the originality of this work is due both to theoretical research and to the solutions that were carried out through the collaboration of the LAS algorithm and the Mathematica program. Therefore, it can be applied "without limits" in all cases of geotechnical problems (especially since the soil displays great heterogeneity), as well as more generally in all the problems that appear in the branch of civil engineering.

9. Conclusions

1) The theoretical background and methods of analysis of the reliability of structures

have evolved considerably in recent years. These developments, coupled with more accurate quantification of uncertainties associated with load and strength of construction, have stimulated interest in the probabilistic view of structures.

2) The reliability of a construction or the probability of its failure is a determining factor in the design process, after considering the possibility of successfully fulfilling its design requirements. Reliability analysis leads to additional security measures that the design engineer should take into account, due to the above-mentioned uncertainties.

3) Studies on metaheuristic optimization algorithms have shown that they can deliver appreciable results in geotechnical applications. It is estimated that their wider application to practical and theoretical geotechnical problems can bring beneficial results and become a particularly useful tool in the hands of civil engineers.

4) The inclusion of failure volume and consequence, to the analysis, has led to an evaluation of risk that can be incorporated into the design process.

5) As the standard dispersion ratio increases, the values of the random fields of the shear strength parameters of the soil become more spread-out.

6) The LAS methodology achieves the objective of creating random property fields with desired quantitative attributes since the comparison of targeted and reached rates is very satisfactory for the average values and the correlation coefficients and quite satisfactory for the typical dispersions.

7) The research undertaken has demonstrated the usefulness of creating random fields variables (random fields) and the LAS method (Local Average Subdivision) in geotechnical engineering and in particular the stability of slopes and leads to the identification of the range of variation of permanent seismic movements.

10. Recommendations

With a view to further analyse, this research can be extended to the following directions:

1) Significant expansion of parametric analyses regarding the strength of the soil, the autocorrelation lengths, as well as the correlation coefficients, so that the analytical results may be used for the creation of weightings in conventional analyses, taking into rational account the spatial variability of soil properties.

2) The use of random fields with the help of the LAS algorithm for 3D probabilistic analysis

Thus, the scope of this research can be broadened and proven to be beneficial towards solving a greater variety of problems arising in the field of Civil Engineering.

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