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# Potential Dependent Frictional Schrodinger Equation

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**Abstract**

By treating particles as harmonic oscillator is obtained the friction energy related to the momentum. The energy and the corresponding Newtonian operator is found. This result in a new Schrodinger equation accounting for the effect of friction. This new equation shows that the energy and mass are quantized, if one treats particles as strings. The radioactive decay law and collision probability is also derived.

***Key words:*** friction; string; harmonic oscillator; radioactive decay law; collision probability.

## 1. Introduction

Quantum mechanics include two independent formulations. The first formulation, called matrix mechanics, was developed by Heisenberg (1925) to describe atomic structure starting from the observed spectral lines of atoms. Heisenberg founded his theory on the notion that the only allowed values of photon are due to the transition of electrons between energy levels of atoms as discrete quanta. Expressing dynamical quantities such as energy, position, momentum and angular momentum in terms of matrices, he obtained an eigenvalue that describes the dynamics of microscopic systems; the diagonalization of the Hamiltonian matrix yields the energy spectrum and the state vectors of the system. Matrix mechanics was very successful in accounting for the discrete quanta of light emitted and absorbed by atoms.The second formulation, called wave mechanics, was due to Schrödinger (1926). It is a generalization of the de Broglie's postulate.

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De Broglie assumes that particles like electrons behave as waves. This method, describes the dynamics of microscopic matter by means of a wave equation, called the Schrodinger equation. Schrodinger equation describes successfully the behavior of atoms. Despite the remarkable successes of quantum equations, but they suffer from noticeable setbacks. For example, the quantum equation cannot differentiate between the behavior of two particles subjected to the same potential, but one moves in free space and the other moves inside matter. This is in direct conflict whit experimental observations [1].This is since the particle move in a media is affected by fraction. Friction is the force resisting the relative motion of solid surfaces, fluid layers, as an example of friction; we can consider a body moving rapidly against a stationary background. Its kinetic energy is dissipated, generating heat and entropy in the environment, the amount of dissipation is proportional to the velocity [2]. The fraction is observed in micro-mechanical systems at low temperatures, in superfluid theory, and even in quantum cosmology [3]. In particular, friction is the term widely used in descriptions of ion collisions. Frictional forces depend in this case on position and their range is comparable with the nuclear radius [4]. Thus, this work is concerned with deriving Schrodinger equation for quantum system suffering from fraction. This is done in section 2. Applications for harmonic oscillator and radioactive decay low are in sections (3) and (4) respectively [5, 6, 7].

## 2. Schrodinger equation for frictional medium

According to Plank and de Broglie hypothesis the quantum quanta are treated as wave packets.

Pure waves is a wave packet consisting of single wave having specific wave length .while a localized particle is a wave packet having a very large of interfering waves having different wave lengths . This means that any quantum system is a single or aggregate of oscillators. Moreover, according to string theory matter building blocks are treated as vibrating string. Motivated by all there hypothesis, the energy dissipated by fraction can be derived consider now a fractional force $\mathrm{F_f}$ in terms of mass m, relaxation time $\tau$ and velocity υ to be

$$\mathrm{E_f} = \frac{\mathrm{mv}}{\tau} \qquad (1)$$

Considering matter building blocks as oscillators

$$\upsilon = \upsilon_{\circ} \mathrm{e}^{\mathrm{iwt}} \qquad (2)$$

Thus, the displacement is given by:

$$x = \int v dt = v_o \int e^{iwt} dt = \frac{v_o}{iw} e^{iwt} = \frac{v}{iw} \qquad (3)$$

The total dissipative energy $E_f$ is given by:

$$\mathrm{E_f} = \int F_f . dx = \frac{\mathrm{m}}{\mathrm{iw}\tau} \int v dv = \frac{mv^2}{2iw\tau} = \frac{imv^2}{2i^2 \tau w} = \frac{-imv^2}{2w\tau} = -\frac{i}{w\tau}\left(\frac{1}{2}mv^2\right) = \frac{-i}{w\tau}\left(\frac{P^2}{2m}\right) \qquad (4)$$

But according to Newtonian mechanics the total energy can be expressed in terms of the kinetic and potential energy V in the form

$$E = K + V = \frac{P^2}{2M} + V \tag{5}$$

Thus according to Eq. (5) and Eq. (4) $E_f$ is given by

$$E_f = \frac{-i}{w\tau}(\mathrm{E} - \mathrm{V}) \tag{6}$$

But using plank hypothesis the energy E is given by:

$$E = \hbar\omega \tag{7}$$

In view of Eqs. (6) and (7) the frictional energy is given by

$$\mathrm{E_f} = \frac{-i\hbar}{\hbar w\tau}(\mathrm{E} - \mathrm{V}) = \frac{i\hbar}{\tau E}(\mathrm{V} - \mathrm{E})$$

$$\mathrm{E_f} = \frac{i\hbar}{\tau}\left(\frac{\mathrm{V}}{E} - 1\right) \tag{8}$$

Thus the Hamiltonian classical relation for a particle in a fractional medium is given by

$$E = H = \frac{P^2}{2m} + V + \frac{i\hbar}{\tau}\left(\frac{V}{E} - 1\right) = \frac{P^2}{2m} + V + \frac{i\hbar}{\tau}\left(\frac{V-E}{E}\right) \tag{9}$$

Therefore

$$E^2 = \left(\frac{P^2}{2m} + V\right)E + \frac{i\hbar}{\tau}(V - E) \tag{10}$$

To find the Schrodinger equation corresponding to this relation multiplies both sides of Eq. (10) by $\Psi$ to get:

$$E^2\Psi = \left(\frac{P^2}{2m} + V\right)E\Psi + \frac{i\hbar}{\tau}(V - E)\Psi \tag{11}$$

Considering the wave function

$$\Psi = Ae^{\frac{i}{\hbar}\ (px - Et)} \tag{12}$$

Hence

$$\frac{\partial\Psi}{\partial t} = -\frac{i}{\hbar}\mathrm{E}\Psi$$

$$E\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\frac{\partial^2\Psi}{\partial t^2} = -\frac{i^2}{\hbar^2}E^2\Psi \tag{13}$$

$$-\hbar^2\frac{\partial^2\Psi}{\partial t^2} = E^2\Psi \tag{14}$$

Similarly differentiating the wave function respect to x yields

$$\frac{\partial\Psi}{\partial x} = \frac{i}{\hbar}\mathrm{P}\Psi$$

$$i\hbar\frac{\partial\Psi}{\partial t} = \mathrm{P}\Psi$$

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{i^2}{\hbar^2}P^2\Psi$$

$$-\hbar^2\frac{\partial^2\Psi}{\partial x^2} = -\hbar^2\nabla^2\Psi = P^2\Psi \tag{15}$$

Thus inserting Eqs. (13), (14) and (15) into Eq. (11) yields

$$-\hbar^2\frac{\partial^2\Psi}{\partial x^2} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)i\hbar\frac{\partial\Psi}{\partial t} + \frac{i\hbar}{\tau}\left(-i\hbar\frac{\partial\Psi}{\partial t} + V\Psi\right)$$

$$-\hbar^2\frac{\partial^2\Psi}{\partial x^2} = i\hbar\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\frac{\partial\Psi}{\partial t} + \frac{\hbar^2}{\tau}\frac{\partial\Psi}{\partial t} + \frac{i\hbar}{\tau}V\Psi \tag{16}$$

## 3. Harmonic oscillator solution

To see how fraction force consider the solution of Eq. (12) in the form

$$\Psi = e^{-i\frac{E}{\hbar}t}u(v) = f(t)u(v) = fu$$

$$\frac{\partial\Psi}{\partial t} = -i\frac{E}{\hbar}fu$$

$$\frac{\partial^2\Psi}{\partial x^2} = \frac{i^2E^2}{\hbar^2}fu = -\frac{E^2}{\hbar^2}fu \tag{17}$$

A direct substitution in Eq. (16) gives

$$E^2fu = i\hbar\left(-\frac{\hbar^2}{2m}\nabla^2u + Vu\right)f\left(\frac{-iE}{\hbar}\right) - i\frac{E\hbar^2}{\hbar\tau}fu + i\frac{\hbar}{\tau}Vfu \tag{18}$$

Dividing both sides of Eq. (18) by f yields

$$E^2 u = +E\left(-\frac{\hbar^2}{2m}\nabla^2 u + Vu\right) - i\frac{E\hbar}{\tau}u + i\frac{\hbar}{\tau}Vu \tag{19}$$

Dividing both sides of Eq. (19) by +E yields

$$\left(E + \frac{i\hbar}{\tau}\right)u = -\frac{\hbar^2}{2m}\nabla^2 u + V\left(1 + \frac{i\hbar}{\tau E}\right)u$$

$$-\frac{\hbar^2}{2m}\nabla^2 u + c_1 V u = E_1 u \tag{20}$$

Where

$$c_1 = 1 + \frac{i\hbar}{\tau E}$$

$$E_1 = E + \frac{i\hbar}{\tau} \tag{21}$$

For harmonic oscillator one finds

$$V = \frac{1}{2}kx^2 \tag{22}$$

Thus substituting this expression in Eq. (20) gives

$$-\frac{\hbar^2}{2m}\nabla^2 \mathrm{u} + c_1 \frac{1}{2}\mathrm{k}x^2 = E_1 \mathrm{u} \tag{23}$$

Let now

$$k_o = c_1 k \tag{24}$$

Therefore equation (23) became

$$-\frac{\hbar^2}{2m}\nabla^2 \mathrm{u} + \frac{1}{2}k_o x^2 = E_1 \mathrm{u} \tag{25}$$

Thus substituting Eq. (21) into Eq. (25) gives

$$E_1 = E + \frac{i\hbar}{\tau} = \left(n + \frac{1}{2}\right)\hbar\omega \tag{26}$$

$$E = \left(n + \frac{1}{2}\right)\hbar\omega - \frac{i\hbar}{\tau} \tag{27}$$

The frequency is given according to Eq. (24) and Eq. (21) to be

$$k_o = m\omega^2$$

$c_1 k = \left(1 + \frac{i\hbar}{\tau E}\right) k = m\omega^2$

$$\left(E + \frac{i\hbar}{\tau}\right) k = m\omega^2 E \qquad (28)$$

Thus

$$E = \left(\frac{m\omega^2}{k} - 1\right)^{-1} \frac{i\hbar}{\tau} \qquad (29)$$

From (3-12) and (3.13)

$$0 = -\frac{m\omega^2}{k} + \left(n + \frac{1}{2}\right)\hbar\omega$$

$$m = \left(1 + \frac{i}{\tau(n+\frac{1}{2})}\right)\frac{k}{\omega^2} \qquad (30)$$

Thus, from Eq. (30) one finds the mass is quantized

## 4. radioactive decay low and collision probability

Consider now Eq. (16) for constant potential $V_o$

Using the separation of variables let the wave function $\Psi$ be in the form

$$\Psi(r,t) = f(t)u(r) = fu \qquad (31)$$

A direct substitution of equation (31) in equation (16) gives

$$-\hbar^2 u \frac{\partial^2 f}{\partial t^2} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_o\right) u \left(i\hbar \frac{\partial f}{\partial t}\right) + \frac{i\hbar}{\tau} V_o u f + \frac{\hbar^2}{\tau} u \frac{\partial f}{\partial t}$$

Thus

$$\left(-\hbar^2 \frac{\partial^2 f}{\partial t^2} - \frac{i\hbar}{\tau} V_o f - \frac{\hbar^2}{\tau}\frac{\partial f}{\partial t}\right) u = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_o\right) u \left(i\hbar \frac{\partial f}{\partial t}\right) \qquad (32)$$

Divide both sides of Eq. (32) by fu to get

$$\left(i\hbar \frac{\partial f}{\partial t}\right)^{-1} \left(-\hbar^2 \frac{\partial^2 f}{\partial t^2} - \frac{iV_o\hbar}{\tau} f - \frac{\hbar^2}{\tau}\frac{\partial f}{\partial t}\right) = \frac{1}{u}\left(-\frac{\hbar^2}{2m}\nabla^2 + V_o\right) u = E_o \qquad (33)$$

Taking the time part of Eq. (33) only gives

$$-\hbar^2 \frac{\partial^2 f}{\partial t^2} - \frac{iV_o\hbar}{\tau} f - \frac{\hbar^2}{\tau}\frac{\partial f}{\partial t} = i\hbar E_o \frac{\partial f}{\partial t} \qquad (34)$$

Consider the case when the potential vanishes

$$V_o = 0 \quad (35)$$

Hence

$$-\hbar^2 \frac{\partial^2 f}{\partial t^2} - \frac{\hbar^2}{t}\frac{\partial f}{\partial t} = i\hbar E_o \frac{\partial f}{\partial t} \quad (36)$$

Consider now a solution

$$f = Ae^{-\frac{i}{\hbar}Et}$$

$$\frac{\partial f}{\partial t} = \frac{-i}{\hbar}Ef$$

$$\frac{\partial^2 f}{\partial t^2} = +\frac{i^2}{\hbar^2}E^2 f = -\frac{E^2}{\hbar^2}f \quad (37)$$

Inserting Eq. (37) in Eq. (36) yields

$$E^2 f + \frac{i\hbar}{\tau}Ef = i\hbar E_o\left(-\frac{i}{\hbar}Ef\right) \quad (38)$$

Dividing both sides of Eq. (38) by f gives

$$E^2 + \frac{i\hbar}{\tau}E = E_o E \quad (39)$$

Rearranging both sides of Eq. (39) gives

$$E^2 = \left(E_o - \frac{i\hbar}{\tau}\right)E \quad (40)$$

Dividing both sides of Eq. (4) by E gives

$$E = \left(E_o - \frac{i\hbar}{\tau}\right) \quad (41)$$

Inserting Eq. (41) in Eq. (47) gives

$$f = Ae^{\frac{-i}{\hbar}\left(E_o - \frac{i\hbar}{\tau}\right)t} = Ae^{-\frac{t}{\tau}}e^{-\frac{i}{\hbar}E_o t}$$

Hence

$$f = Ae^{\frac{-t}{\tau}}e^{-\frac{i}{\hbar}E_o t} \quad (42)$$

Since the probability and number of particles are given by

$$n = |f|^2 = ff^{\backslash} = A^2 e^{\frac{-2t}{\tau}} \tag{43}$$

Eq. (43) is the ordinary radioactive decay low with

$$\lambda = \frac{2}{\tau} \quad , \quad n_o = A^2 \tag{44}$$

i.e.

$$n = n_o e^{-\lambda t} \tag{45}$$

This expression also gives collision probability p with

$p = n \qquad p_o = A^2$

$$\tau_o = \tau/2 \tag{46}$$

To get

$$p = p_o\, e^{\frac{-t}{\tau_o}} \tag{47}$$

Eq. (47) is the ordinary collision probability relation.

## 5. Discussion

New Schrodinger equation for frictional medium was derived by using de Broglie hypothesis about the wave nature of atomic particles, besides assuming that particles are vibrating strings.

The two hypotheses require that for frictional medium the classical energy is that of a harmonic oscillator [see Eqs. (2), (3), (4) and (5)]

Using canonical quantization method by replacing the momentum and energy with their corresponding operators, [see Eqs. (12), (13) and (14)], a new frictional Schrodinger equation was derived [see Eq. (15) ].

It is very interesting to note that using separation of variables the time dependent part was used to derive radioactive decay law as shown by Eq. (43) and the collision probability as shown by Eq. (47).

Solving for harmonic oscillator the expression for energy is quantized with additional frictional term [see Eq. (27)].

The solution of Eq. (27) shows that the mass is quantized.

## 6. Conclusion

The Schrodinger equation for frictional medium shows that the energy and mass are quantized. It is also used to derive radioactive decay low and collision probability.

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