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A Method of Finding the Distance between Two Places on the Surface of the Earth

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Abstract

Differential Geometry is the language of modern physics and provides us with a new technique of solving the problems related to the calculation of the plane and surface with parameters. There is a lot of application of Differential Geometry in calculating some of the geographical problems. Finding the distance between my house to my friend's house in the next block is very easy but the approach to calculate the distance between two places holds true only to a certain extent. In this paper we have tried to build a method for finding the distance between two places on the surface of the earth using differential Geometry.

Keywords: Great circle; Sphere; Latitude and Longitude.

1. Introduction

It is imagined that the earth is a perfect sphere with an axis around which it spins. The end of the axis are the North and South Pole. Every places of the earth has its latitude and longitude which are considered as the arcs of the circles. The shortest distance between two places that covered by great circle as geodesics on a sphere are the arcs of great circle [5]. With the help of spherical trigonometry and by using latitude and longitude we can calculate the distance and direction from one place to another on the surface of the earth with respect to the geographical north pole or south pole.

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1.1. Sphere

A sphere is a solid figure such that every point of its surface is equally distant from a fixed point with in it, which is called center of the sphere. The straight line drawn through the center and terminated both ways by the sphere is called a diameter and any straight line joining the center of sphere to any point on the surface is called a radius of the sphere.

1.2. Theorem

1.3. If the arc S of a circle with radius r subtends an angles θ at the center then $S = r\theta$ where θ is measured in radians

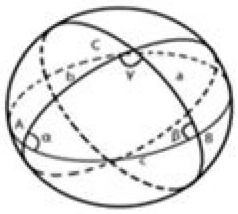

Figure 1

2. Spherical Triangle

A spherical triangle [3] is the proportion of a sphere bounded by three arcs of great circles. The arcs are its sides and spherical angles between the arcs are its three angles.

2.1. Cosine Formula

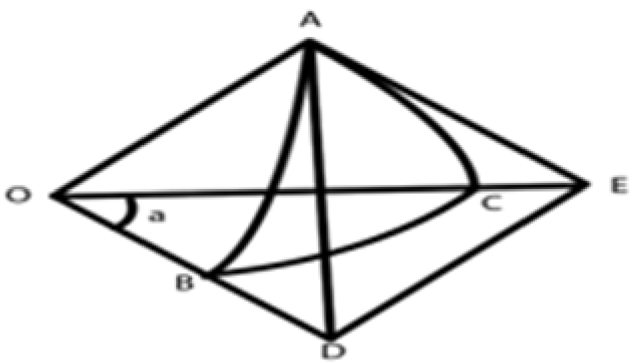

Figure 2

To find the value of cosine of an angle of a spherical triangle in terms of cosines and sines of the sides [3].

ABC is spherical triangle, O the center of sphere. Draw two tangents at A to the arc AB and AC which intersect the lines OB and OC extended to D and E respectively.

Join DE. Then $\angle A = \angle DAE$.

Let $\angle DOE = a$. Now, in the triangle *DOE*

$$DE^2 = OD^2 + OE^2 - 2OD.OE.\cos DOE \qquad (1)$$

Again in the triangle *DAE*

$$DE^2 = AD^2 + AE^2 - 2AD.AE.\cos DAE \qquad (2)$$

Subtracting (2) from (1)

$$0 = OD^2 - AD^2 + OE^2 - AE^2 + 2AD.AE.\cos A - 2OD.OE.\cos a$$

Also the angles *OAD* and *OAE* are right angles, so that $OD^2 = OA^2 + AD^2$ and $OE^2 = OA^2 + AE^2$.

Hence we have

$$0 = OA^2 + OA^2 + 2AD.AE.\cos A - 2OD.OE.\cos a$$

Changing sides

$$\cos a = \frac{OA}{OD} \cdot \frac{OA}{OE} + \frac{AD}{OD} \cdot \frac{AE}{OE} \cos a$$

Therefore

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Similarly we can prove that

$$\cos b = \cos c \cos a + \sin c \sin a \cos B$$

and $$\cos c = \cos a \cos b + \sin a \sin b \cos C \ .$$

2.2. Sine Formula

To prove that the angles of a spherical triangle are proportional to the sines of the opposite sides [3].

That is,

Figure 3

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Proof. We already proved in cosine formula that

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$$

$$\therefore\ \sin^2 A = 1 - \cos^2 A = 1 - \frac{(\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c}$$

$$or,\ \sin^2 A = \frac{(1-\cos^2 b)(1-\cos^2 c) - (\cos^2 a + \cos^2 b \cos^2 c - 2\cos a \cos b \cos c)}{\sin^2 b \sin^2 c}$$

$$or,\ \ \sin^2 A = \frac{(1-\cos^2 b - \cos^2 c + \cos^2 b \cos^2 c) - (\cos^2 a + \cos^2 b \cos^2 c - 2\cos a \cos b \cos c)}{\sin^2 b \sin^2 c}$$

$$or, \sin A = \frac{\sqrt{(1-\cos^2 a - \cos^2 b + \cos^2 c + 2\cos a \cos b \cos c)}}{\sin b \sin c}$$

We have taken only positive sign, with the radical as we know that the angle and sides of a spherical triangle are each less than two right angles and such $\sin A$, $\sin B$, $\sin C$ are all positive.

$$\therefore \frac{\sin A}{\sin a} = \frac{\sqrt{(1-\cos^2 a - \cos^2 b + \cos^2 c + 2\cos a \cos b \cos c)}}{\sin a \sin b \sin c}$$

The symmetry of the result shows that

$$\therefore \frac{\sin B}{\sin b} = \frac{\sqrt{(1-\cos^2 a - \cos^2 b + \cos^2 c + 2\cos a \cos b \cos c)}}{\sin a \sin b \sin c}$$

$$and\ \ \frac{\sin C}{\sin c} = \frac{\sqrt{(1-\cos^2 a - \cos^2 b + \cos^2 c + 2\cos a \cos b \cos c)}}{\sin a \sin b \sin c}$$

So we have

$$\frac{\sin A}{\sin a}=\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c}=\frac{\sqrt{(1-\cos^2 a-\cos^2 b+\cos^2 c+2\cos a\cos b\cos c)}}{\sin a\sin b\sin c}$$

$$\therefore\ \frac{\sin A}{\sin a}=\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c}$$

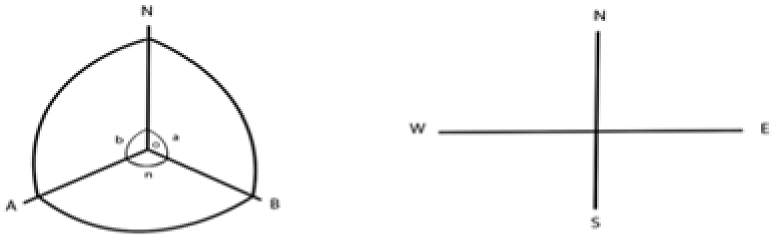

Figure 4

3. Shortest Distance and Direction Between Places

Let NAB be the spherical triangle where A and B the two places on the sphere of the earth. $A \equiv (x_1°N\ ,\ y_1°E)$ $B \equiv (x_2°N\ , y_2°E)$.

N be the north pole. "O" be the center of the sphere of the earth. Let, $\angle AOB = n,\ \angle BON = a,\ \angle AON = b$. We have to find out the shortest distance between the given two places and the direction of the two places towards North pole.

3.1. Calculation for the shortest distance between A and B towards North pole

We have from cosine rule of spherical trigonometry

$$\cos n = \cos a \cos b + \sin a \sin b \cos N$$

$$= \cos(90° - \mathrm{x}_2°)\cos(90° - \mathrm{x}_2°) + \sin(90° - \mathrm{x}_2°)\sin(90° - \mathrm{x}_2°)\cos(\mathrm{y}_2° - \mathrm{y}_1°)$$
$$= \sin \mathrm{x}_2° \sin \mathrm{x}_1° + \cos \mathrm{x}_2° \cos \mathrm{x}_1° \cos(\mathrm{y}_2° - \mathrm{y}_1°)$$

$$\therefore\ n = \cos^{-1}\{\sin \mathrm{x}_2° \sin \mathrm{x}_1° + \cos \mathrm{x}_2° \cos \mathrm{x}_1° \cos(\mathrm{y}_2° - \mathrm{y}_1°)\} \qquad (1)$$

$\therefore$ The distance between A and B = the length of the arc of the great circle passing through A and $B = d = R.n$ (R =Radius of the earth).

By equation (1) we can easily find out the value of n for any two places and hence we can find out shortest distance d.

3.2. Calculation for the direction

From spherical triangle using sine rule

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin N}{\sin n}$$

$$\text{or,}\quad \sin A = \frac{\sin a \sin N}{\sin n}$$

$$\text{or,}\quad \sin A = \frac{\sin(90° - \mathrm{x}_2°)\sin(\mathrm{y}_2° - \mathrm{y}_1°)}{\sin n}$$

$$\text{or,}\quad \sin A = \frac{\cos \mathrm{x}_2° \sin(\mathrm{y}_2° - \mathrm{y}_1°)}{\sin n} \qquad (2)$$

Similarly,

$$\sin B = \frac{\cos \mathrm{x}_1° \sin(\mathrm{y}_2° - \mathrm{y}_1°)}{\sin n} \qquad (3)$$

By, these two equations we can easily find out the direction of any place with respect to another place towards North Pole.

4. Examples

4.1. Example

To find the shortest distance and direction from one towards the other between ***London*** (U.K) and ***Bangkok*** (Thailand) towards North Pole.

Here, $A \equiv London;$ $x_1 = 51.30°N$

$y_1 = 0.10°W$

$B \equiv Bangkok;$ $x_2 = 13.44°N$

$y_2 = 100.30°E$

$$\mathrm{N} = \mathrm{y}_2° - \mathrm{y}_1° = 100.30\,° - (-0.10°)$$

$$= 100.30° + 0.10° = 100.40°$$

We know from Spherical Trigonometry,

$$n = \cos^{-1}\{\sin \mathrm{x}_2° \sin \mathrm{x}_1° + \cos \mathrm{x}_2° \cos \mathrm{x}_1° \cos(\mathrm{y}_2° - \mathrm{y}_1°)\}$$

$$= \cos^{-1}\{\sin 51.30° \sin 13.44° + \cos 51.30° \cos 13.44° \cos 100.40\}$$

$$= \cos^{-1}(0.1813931 - 0.1097772)$$

$$= \cos^{-1}(0.0716159)$$

$$= 85.8931980°$$

$$= 85.8931980 \times \frac{\pi}{180}\ \ radian$$

$$= 1.4991191 radian$$

<u>Calculation for the shortest distance:</u>

Radius of the earth, $R = 6378.388\ \ kms$

∴ Shortest Distance between $London$ and $Bangkok$ is,

$$d = R \cdot n$$

$$= 6378.388 \times 1.4991191 = 9561.96\ \ kms$$

So, the distance between London and Bangkok is $9561.96\ \ kms$.

<u>Calculation for the direction of London and Bangkok towards North Pole:</u>

From the *sine* rule on spherical triangle we have,

$$\sin A = \frac{\cos \mathrm{x}_2° \sin(\mathrm{y}_2° - \mathrm{y}_1°)}{\sin n}$$

$$= \frac{\cos 13.44° \sin 100.40°}{\sin 85.8931980°}$$

$$= 0.9590979$$

$$\therefore \angle A = \sin^{-1}(0.9590979)$$

$$= 73.56°$$

Again, $$\sin B = \frac{\cos \mathrm{x}_1° \sin(\mathrm{y}_2° - \mathrm{y}_1°)}{\sin n}$$

$$= \frac{\cos 51.30° \sin 100.40°}{\sin 85.8931980\,°}$$

$$= 0.6165539$$

$$\therefore \angle B = \sin^{-1}(0.6165539)$$

$$= 38.07°$$

The direction of Bangkok with respect to London towards graphical north pole is

$$\angle A = 73.56°$$

The direction of London with respect to Bangkok towards graphical north pole is

$$\angle B = 38.07°$$

4.2. Example

To find the shortest distance and direction for ***Dhaka*** and ***Munshiganj*** towards North Pole.

Here, $A \equiv Dhaka;$ $x_1 = 23.42°N$

$y_1 = 90.22°E$

$B \equiv Munshigang;$ $x_2 = 23.32°N$

$y_2 = 90.32°E$

$$\mathrm{N} = \mathrm{y_2}° - \mathrm{y_1}° = 90.32° - 90.22° = 0.10°$$

We know from Spherical Trigonometry,

$$n = \cos^{-1}\{\sin \mathrm{x_2}° \sin \mathrm{x_1}° + \cos \mathrm{x_2}° \cos \mathrm{x_1}° \cos(\mathrm{y_2}° - \mathrm{y_1}°)\}$$

$$= \cos^{-1}\{\sin 23.42° \sin 23.32° + \cos 23.42° \cos 23.32° \cos(90.32° - 90.22°)\}$$

$$= \cos^{-1}(0.157344 - 0.842653)$$

$$= \cos^{-1}(0.999997)$$

$$= 0.1403345°$$

$$= 0.1403345 \times \frac{\pi}{180}\ radian$$

$$= 0.002449\ radian$$

<u>Calculation for the shortest distance:</u>

Radius of the earth [3], $R = 6378.388\ kms$

$\therefore$ Shortest Distance between Dhaka and Munshiganj is,

$$d = R \cdot n$$

$$= 6378.388 \times 0.002449$$

$$= 15.6237\ kms$$

So, the distance between Dhaka and Munshiganj is $15.6237\ kms$.

Calculation for the direction of Dhaka and Munshiganj towards North Pole:

From the *sine* rule on spherical triangle we have,

$$\sin A = \frac{\cos \mathrm{x}_2° \sin(\mathrm{y}_2° - \mathrm{y}_1°)}{\sin n}$$

$$= \frac{\cos 23.32° \sin(0.10°)}{\sin 0.140345°} = 0.654553$$

$$\therefore \angle A = \sin^{-1}(0.654553)$$

$$= 40.89°$$

Again, $\sin B = \frac{\cos \mathrm{x}_1° \sin(\mathrm{y}_2° - \mathrm{y}_1°)}{\sin n}$

$$= \frac{\cos 23.42° \sin(0.10°)}{\sin 0.140345°}$$

$$= 0.653829$$

$$\therefore \angle B = \sin^{-1}(0.653829) = 40.83°$$

The direction of Munshiganj with respect to Dhaka towards graphical north pole is

$$\angle A = 40.89°$$

The direction of Dhaka with respect to Munshiganj towards graphical north pole is

$$\angle B = 40.83°$$

5. Conclusion

Differential Geometry is related to Astronomy and Geographical problems. Some practical examples are discussed in this paper. The aim of this paper is to find a mathematical method of finding distance and direction between two places on the surfaces of the earth with respect to the geographical north pole or south pole. Since the radius of the earth is not perfect, so the distance and directions found here are approximate.

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