

Synthesis of Antenna Arrays for Maximum Gain and Its Impact on BER Performance of MIMO Systems

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Abstract

Multiple-Input Multiple-Output (MIMO) is the most promising technology that improves the system capacity and data rate by using multiple antennas at transmitting and receiving sides of wireless communication systems. Many research efforts are introduced to enhance the bit error rate (BER) performance of MIMO systems. In this paper, detection algorithms based on antenna arrays synthesis have been proposed for bit error rate performance enhancement of MIMO systems. It is well known that the maximum number of data streams that can be supported by MIMO system using spatial multiplexing is given by $N_{stream} = \min(N_T, N_R)$. Where N_T is the number of transmitting antennas and N_R is the number of receiving antennas. For a given $(N_T \times N_R)$ MIMO system, the existing number of antenna elements at transmitting and receiving sides are individually used to synthesize larger size antenna arrays to provide higher antenna gains without using additional antenna elements or changing the number of transmitted data streams. The achieved array gain will enhance the signal to noise ratio of the system giving rise to lower bit error rate. The proposed system is tested for both Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) detectors. The simulation results revealed that the proposed system outperforms the traditional ZF and MMSE detectors.

Keywords: Multi Input- Multi Output (MIMO); Zero Forcing (ZF); Minimum Mean Square Error (MMSE); Bit Error Rate (BER).

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1. Introduction

Multiple-input multiple-output (MIMO) systems exploit multipath propagation to achieve higher data rates, without the need for additional bandwidth [1-5]. For optimum detection, the Maximum Likelihood accomplishes the most elevated performance to the detriment of high computational complexity.

In this context, great research efforts are exerted to introduce new detection algorithms of high performance close to the ML optimal performance.

The sphere decoding (SD) algorithm can efficiently attain the optimal maximum-likelihood (ML) performance, although it has higher complexity at lower signal-to-noise ratio (SNR) [2,3]. The linear detectors, like the Minimum Mean Square Error (MMSE) based detectors, have much lower complexity, but there also exists significant performance degradation comparing with the SD algorithm. On the other hand, non-linear detectors provide better performance with higher complexity. The vertical Bell-laboratories layered space-time (V-BLAST) system, is based on the MMSE principle with the optimal successive interference cancellation (OSIC). The V-BLAST is one of the most well-known MIMO detection techniques [4].

A fast detector for MIMO system which utilizes the lattice reduction to improve the symbols detection was introduced in [5]. It employs the orthogonality defect factor to select the best available channel submatrix for conditional detection. After channel submatrix selection, the lattice reduction is applied to the first channel submatrix. Zero Forcing (ZF) as a linear detector is applied to estimate the first symbols subset. These estimated symbols are used in conditional detection for the second symbols subset applying the ML detection technique. The estimated second symbols subset are utilized again to enhance the estimation of the first symbols subset applying the ZF detector again. But, starting with ZF detector which has lower estimation accuracy than ML may affect the results of the second and third stages.

In this paper, enhanced detection techniques based on synthesis of antenna arrays at both transmitting and receiving sides are introduced for BER performance enhancement of MIMO systems. Many research efforts for antennas arrays synthesis using reduced number of antenna elements are introduced in [6-10]. In [10], the efficient MOM/GA array synthesis technique was introduced for arbitrarily shaped patterns synthesis using reduced number of antenna elements. In this work, the reverse process is executed where the existing few number of antenna elements is used to synthesize larger size antenna arrays with higher gains. This synthesis is intended to reduce the BER by increasing the signal to noise ratio of the MIMO signal via increasing the array gain.

2. Problem Formulation

Consider an $N_T \times N_R$ MIMO model as shown in Figure (1) where N_T and N_R are the number of transmitting and receiving antennas respectively. The antenna elements of the transmitting and receiving arrays are aligned linearly with uniform spacing $d = \lambda/2$. The received signal y is given by [1].

$$y = Hx + v \quad (1)$$

where x is the $N_T \times 1$ baseband signal vector transmitted during each symbol period formed by the antenna elements. y denotes the received symbol vector with dimension $N_R \times 1$ where $N_T \leq N_R$. v is a complex white Gaussian noise vector of dimensions $N_R \times 1$ with zero mean and variance σ^2 . The channel matrix H is a $N_R \times N_T$ matrix representing the scattering effects of the channel. For simplicity the channel matrix H is considered to be known at the receiver. The main drawback of this system is that the utilized linear antenna arrays suffer from their limited array gains. Traditionally to increase the array gain, the array size or number of antenna elements should be increased. Consequently, the number of RF chains, number of data streams, system complexity, and cost are increased. It is required to find a promising solution to achieve higher array gains without changing the RF front end structure of the existing MIMO system.

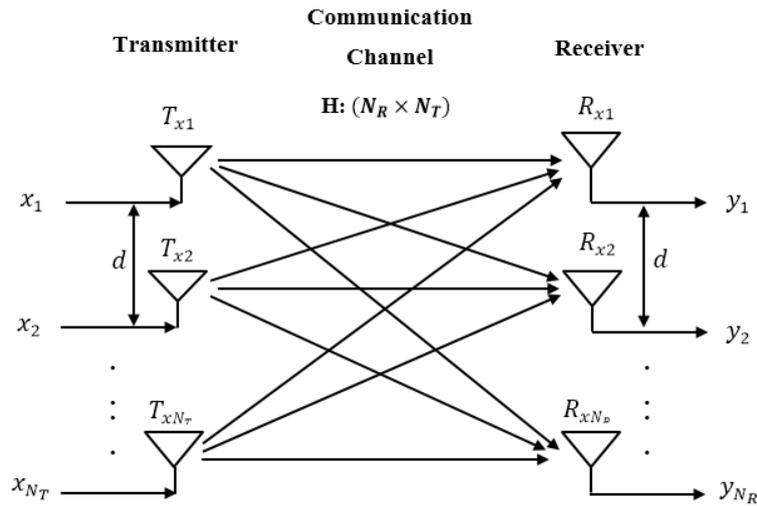


Figure 1: Traditional $N_T \times N_R$ MIMO system model.

3. Proposed MIMO Signal Model

The traditional MIMO system employs uniform linear antenna arrays at both transmitting and receiving sides as shown in Figure (1). To mitigate the aforementioned problems of the traditional MIMO, The limited gain ULA is replaced by a synthesized higher gain non-uniform feeding linear antenna array using the same number of antenna elements, N_T and N_R , and the same number of data streams which is given by:

$$N_{stream} = \min(N_T, N_R) \quad (2)$$

When applying array synthesis, the excitation coefficients are no longer uniform. In this case, the MIMO system model can be redrawn as shown in Figure (2). For this purpose, the MOM/GA array synthesis technique introduced in [10] is used to synthesis the radiation pattern of a chosen large size antenna array with reduced number of antenna elements which equals the number of antenna elements of the traditional MIMO system as shown in Figure (2). The original large size array and the synthesized array patterns have almost the same characteristics such as side lobe level (SLL), half power beamwidth (HPBW), array gain, and directivity. The array factor of original large size array $AF(\theta)$ and the array factor of the synthesized array $AF_{syn}(\theta)$ should be

close to each other which can be expressed as follows:

$$AF(\theta) \approx AF_{syn}(\theta) \tag{3}$$

Equation (3) can be written as [10]

$$\sum_{m=0}^{M-1} a_m e^{jkm d \cos(\theta)} \approx \sum_{n=0}^{N_T-1} a_n e^{jkn d_s \cos(\theta)} \tag{4}$$

where M is the number of antenna elements of the large size array and N_T is the number of elements of the synthesized array where $M > N_T$. a_m and a_n are the excitation coefficients of the original and synthesized arrays respectively. d and d_s are the element spacing of the original and synthesized arrays respectively.

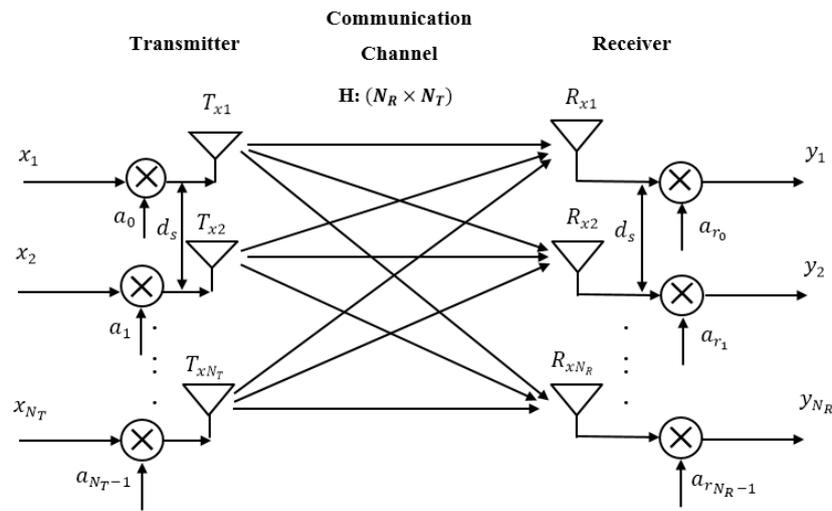


Figure 2: Proposed MIMO system model with synthesized transmitting and receiving antenna arrays.

Consider $(N_T = N_R)$ MIMO system whose antenna arrays are radiating in the broadside direction and aligned together. In this case, the synthesized arrays steering vectors are identical such that $A_t = A_r = A$. Where A_t and A_r are the synthesized steering vectors of the transmitting and receiving antenna arrays respectively. To derive the MIMO signal model, consider applying array synthesis at transmitting side only. Then received signal will be:

$$y = H (A \cdot x) + v \tag{5}$$

where (\cdot) is the dot product. When applying array synthesis at receiving side, the total received signal y_r can be written as:

$$y_r = (H (A \cdot x) + v) \cdot A \tag{6}$$

or

$$y_r = H (A^2 \cdot x) + A \cdot v \tag{7}$$

Let $W = \text{diag}(A^2)$ is a square matrix of dimensions $(N_T \times N_R)$ and $(N_T = N_R)$. Also let $n = A \cdot v$. Then Equation (7) can be written as:

$$y_r = W H x + n \tag{8}$$

For more simplicity let $\Psi = W H$, then Equation (8) is written as follows:

$$y_r = \Psi x + n \tag{9}$$

To express Equation (9) in matrix form, consider the steering vectors $A_t = A_r = A$ which are derived from the synthesized array factor $AF_{syn}(\theta)$ substituting $\theta = \theta_i$. As stated previously, the transmitting and receiving antenna arrays are aligned together in broadside direction, hence $\theta_i = 90^\circ$ with respect to the array line. The steering vectors at broadside direction can be written as follows:

$$A_t = A_r = A = AF_{syn}(90^\circ) \tag{10}$$

$$A_t = A_r = A = [a_0 \ a_1 \ a_2 \ \dots \ \dots \ \dots \ a_{N_T-1}]^T \tag{11}$$

Also, the remaining parameters can be expressed in matrix form as follows

$$x = [x_1(n) \ x_2(n) \ x_3(n) \ \dots \ \dots \ \dots \ x_{N_T}(n)]^T \tag{12}$$

$$x_n(n) = [x_n(1) \ x_n(2) \ x_n(3) \ \dots \ \dots \ \dots \ x_n(N)]^T \tag{13}$$

$$n = [n_1(n) \ n_2(n) \ n_3(n) \ \dots \ \dots \ \dots \ n_{N_T}(n)]^T \tag{14}$$

$$n_n(n) = [n_n(1) \ n_n(2) \ n_n(3) \ \dots \ \dots \ \dots \ n_n(N)]^T \tag{15}$$

$$y_r = [y_1(n) \ y_2(n) \ y_3(n) \ \dots \ \dots \ \dots \ y_{N_R}(n)]^T \tag{16}$$

$$y_n(n) = [y_n(1) \ y_n(2) \ y_n(3) \ \dots \ \dots \ \dots \ y_n(N)]^T \tag{17}$$

where $[]^T$ is the matrix transpose.

4. Simulation Results and Discussions

In order to analyze the impact of the proposed technique on the BER, the 8×8 LTE MIMO system is taken as the simulation object.

It employs two equal size uniform linear antenna arrays at both transmitting and receiving sides with uniform element spacing $d = \lambda/2$. The transmitted signal is modulated using 4-QAM and transmitted over a Rayleigh

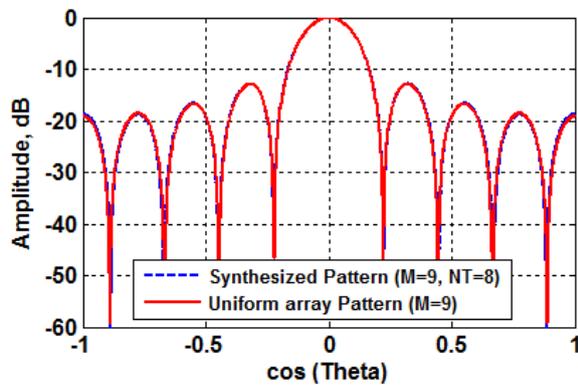
fading channel.

The signal is subjected to an additive white Gaussian noise (AWGN) with zero mean and variance σ^2 . Applying the array synthesis algorithm presented in [10], the dedicated $N_T = N_R = 8$ elements are used to synthesize different larger size antenna arrays from $M = 9$ to $M = 16$ as shown in Figure (3).

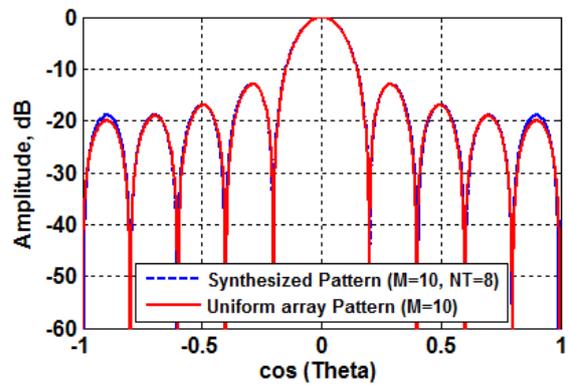
But, as the number of antenna elements M increase, the side lobe level increase.

The grating lobes appear significantly at $M = 15$ and $M = 16$. The synthesized array parameters and excitations are listed in Table (1).

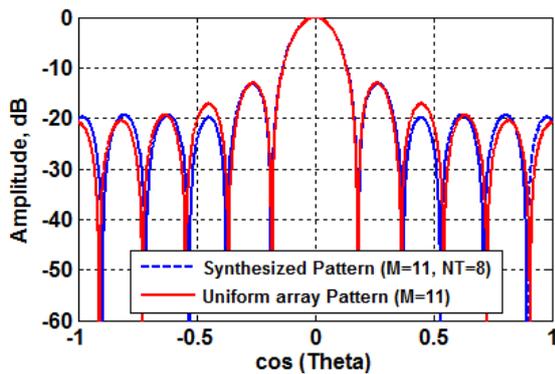
To verify the effectiveness of the proposed technique, two simulation test cases considering ZF and MMSE detectors are presented in the next sections.



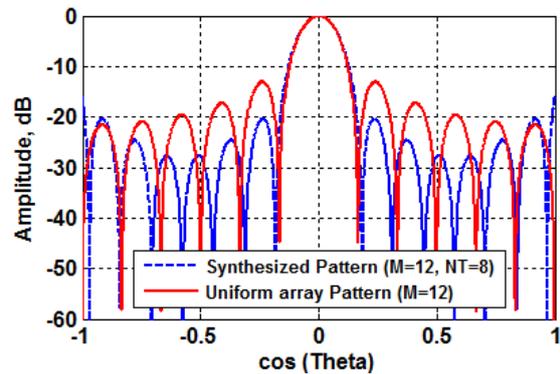
(a) ($M = 9, N_T = N_R = 8$)



(b) ($M = 10, N_T = N_R = 8$)



(c) ($M = 11, N_T = N_R = 8$)



(d) ($M = 12, N_T = N_R = 8$)

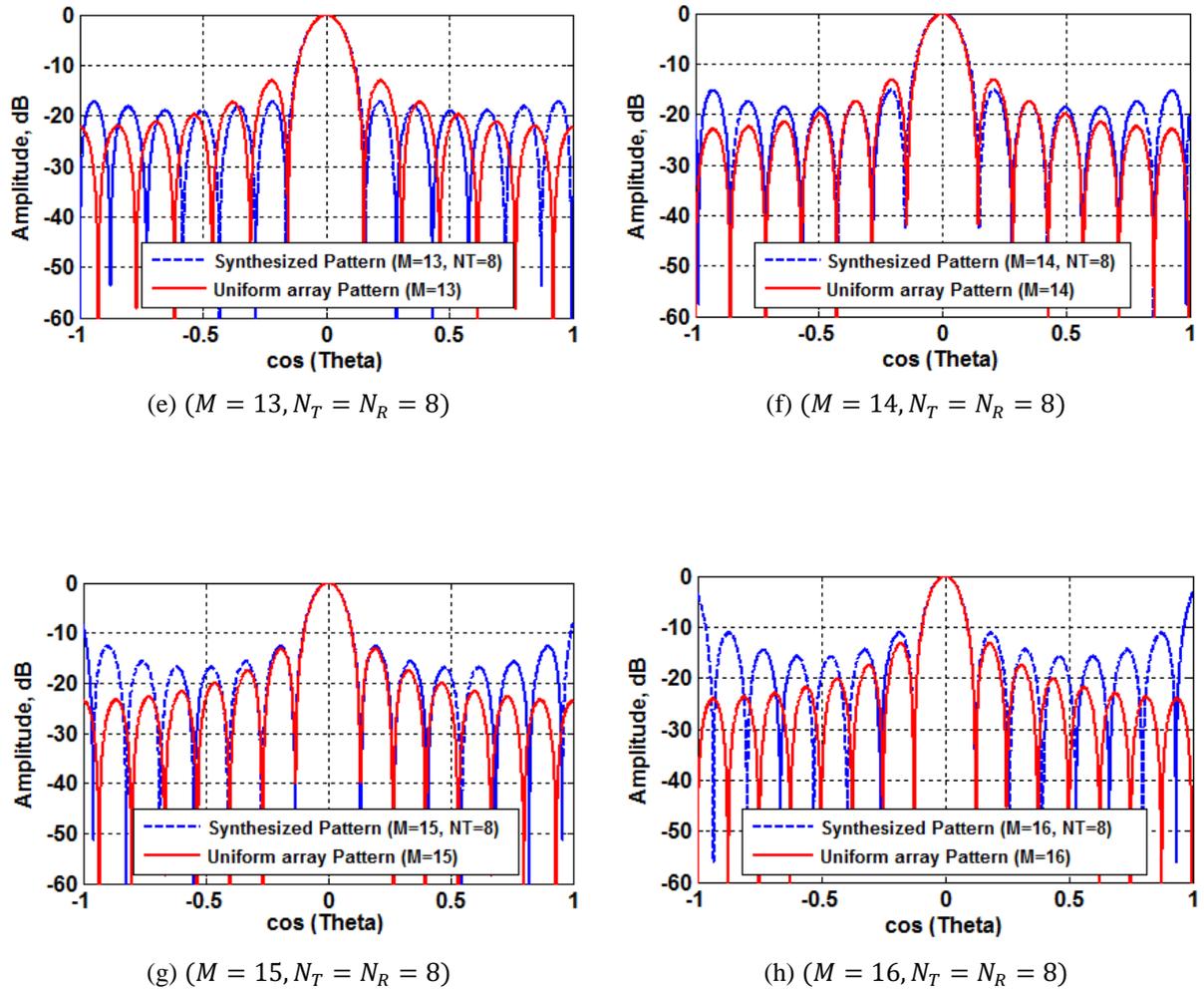


Figure 3: The synthesized antenna arrays using $N_T = N_R = 8$ elements for different larger size antenna arrays from $M = 9$ to $M = 16$.

Table 1: The synthesized array parameters for $M = 9$ to $M = 16$ using $N_T = N_R = 8$ elements

M	9	10	11	12	13	14	15	16
d_s	0.564λ	0.626λ	0.701λ	0.75λ	0.869λ	0.879λ	0.915λ	0.944λ
a_1	1.0892	1.1930	1.2061	1.3415	1.2964	1.4636	1.4198	0.9000
a_2	1.144	1.2673	1.3920	1.4908	1.3678	1.5052	1.3144	0.7752
a_3	1.1196	1.2494	1.4952	1.5509	1.6494	1.6865	1.3236	0.7216
a_4	1.1307	1.2518	1.3518	1.479	1.8438	1.8100	1.3414	0.6999
a_5	1.1307	1.2518	1.3518	1.479	1.8438	1.8100	1.3414	0.6999
a_6	1.1196	1.2494	1.4952	1.5509	1.6494	1.6865	1.3236	0.7216
a_7	1.144	1.2673	1.3920	1.4908	1.3678	1.5052	1.3144	0.7752
a_8	1.0892	1.1930	1.2061	1.3415	1.2964	1.4636	1.4198	0.9000

4.1 ZF Detector Based on Array Synthesis (ZF_{syn})

According to Equation (1), the traditional Zero-Forcing detection as a low complexity linear detection algorithm gives the estimate of x as follows [11]:

$$\hat{x} = H^\dagger y = x + (H^H H)^{-1} H^H v = x + \hat{v}_{zf} \quad (18)$$

The detector thus forces the interference to zero. The matrix H^\dagger nullifying the interference is given by:

$$H^\dagger = (H^H H)^{-1} H^H \quad (19)$$

where H^\dagger is the pseudo inverse of the channel matrix H . Using the proposed signal model expressed in Equation (9), the ZF detection process applying array synthesis can be summarized as follows where the matrix Ψ^\dagger nullifying the interference is given by:

$$\Psi^\dagger = (\Psi^H \Psi)^{-1} \Psi^H \quad (20)$$

where Ψ^\dagger is the pseudo inverse of the matrix Ψ . The new symbols estimates \hat{x}_{syn} will be given as follows:

$$\hat{x}_{syn} = \Psi^\dagger y_r = x + (\Psi^H \Psi)^{-1} \Psi^H n = x + \hat{n}_{zf} \quad (21)$$

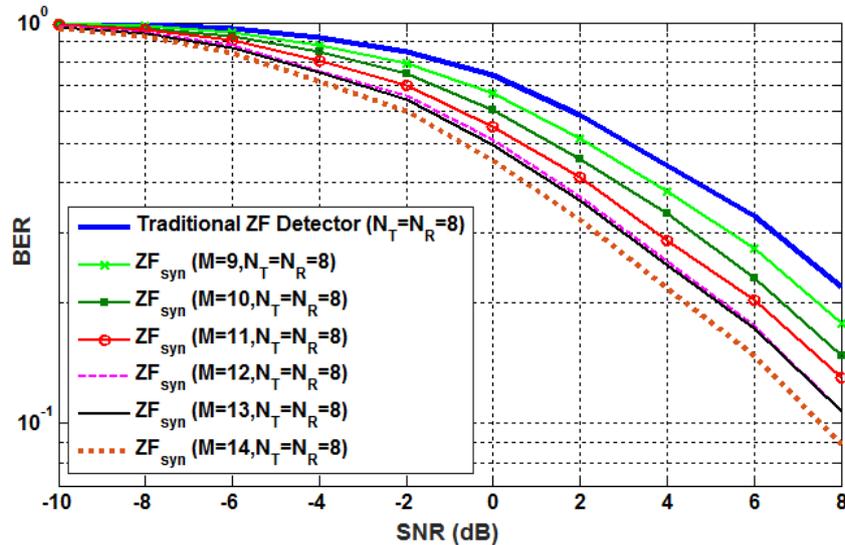


Figure 4: The BER versus SNR for different array sizes ($M > N_T$) synthesized using the same number of antenna elements $N_T = N_R = 8$ compared to the traditional ZF detector in a 8×8 MIMO system.

To verify the effectiveness of the proposed technique, the BER versus signal to noise ratio (SNR) is plotted for the ZF detector applying array synthesis, ZF_{syn} , as shown in Figure (4). Replacing the 8-elements uniform antenna arrays by the 8-elements synthesized antenna arrays, it is found that the BER performance is significantly enhanced as the number of antenna elements increases from $M = 9$ to $M = 14$ as shown in Figure (4). But, for $M = 15$ and $M = 16$, the appearance of grating lobes highly degrades the BER performance as shown in Figure (5). For examples, at $SNR = 0dB$ and $M = 14$, the BER is reduced by 0.2867 compared to the

traditional ZF detector. Also, the simulation results revealed that ($M = 14, N_T = N_R = 8$) system provides the best BER performance.

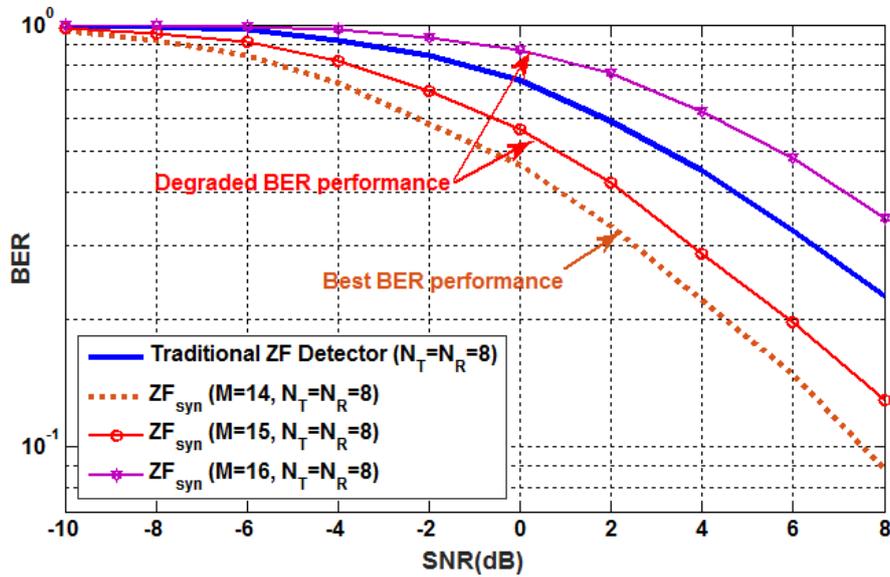


Figure 5: The BER performance degradation of the proposed ZF detector at ($M \geq 15, N_T = N_R = 8$) for a 8×8 MIMO system.

4.2 MMSE Detector Based on Array Synthesis ($MMSE_{syn}$)

According to Equation (1), the MMSE detector estimates the transmitted vector x by applying a linear transformation to the received vector y . It finds out the estimate \hat{x}_{MMSE} of the transmitted symbol vector x as [11]:

$$\begin{aligned} \hat{x}_{MMSE} &= W_{MMSE} y = (H^H H + \sigma^2 I)^{-1} H^H y \\ &= \hat{x} + (H^H H + \sigma^2 I)^{-1} H^H v = \hat{x} + \hat{v}_{MMSE} \end{aligned} \quad (22)$$

The MMSE weight matrix, W_{MMSE} , is utilized to maximize the post-detection signal-to interference plus noise ratio (SINR) [11]. While the MMSE receiver requires the statistical information of noise variance σ^2 . Its BER performance is superior to ZF detection due to mitigating the noise enhancement. The MMSE weight matrix W_{MMSE} is given by:

$$W_{MMSE} = (H^H H + \sigma^2 I)^{-1} H^H \quad (23)$$

Using the proposed signal model expressed in Equation (9), the MMSE detection process applying array synthesis can be summarized as follows where the MMSE weight matrix $W_{MMSE_{syn}}$ is given by:

$$W_{MMSE_{syn}} = (\Psi^H \Psi + \sigma^2 I)^{-1} \Psi^H \quad (24)$$

The new symbols estimates \hat{x}_{syn} will be given as follows:

$$\begin{aligned} \hat{x}_{syn} &= W_{MMSE_{syn}} y_r = (\Psi^H \Psi + \sigma^2 I)^{-1} \Psi^H y_r \\ &= x + (\Psi^H \Psi + \sigma^2 I)^{-1} \Psi^H z = x + \hat{n}_{MMSE} \end{aligned} \quad (25)$$

Also, replacing the 8-elements uniform antenna array by the 8-elements synthesized antenna arrays, it is found that the BER performance of the MMSE detector is significantly enhanced as the number of elements increase from $M = 9$ to $M = 14$ as shown in Figure (6). Also, for $M = 15$ and $M = 16$, the appearance of grating lobes highly degrades the BER performance as shown in Figure (7). For examples, at $SNR = 0dB$ and $M = 14$, the BER is reduced by 0.4257 compared to the traditional MMSE detector. The simulation results revealed that ($M = 14, N_T = 8$) system provides the best BER performance.

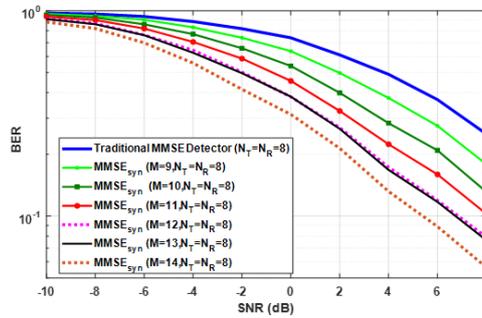


Figure 6: The BER versus SNR for different array sizes ($M > N_T$) synthesized using the same number of antenna elements $N_T = N_R = 8$ compared to the traditional MMSE detector in a 8×8 MIMO system.

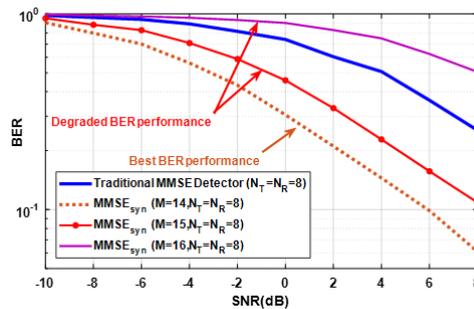


Figure 7: The BER performance degradation of the proposed MMSE detector at ($M \geq 15, N_T = 8$) for a 8×8 MIMO system.

5. Conclusion

In this paper, new detection techniques based on antenna arrays synthesis for maximum gain have been introduced. The achieved array gain significantly enhanced the signal to noise ratio of the system giving rise to better BER performance. To verify the effectiveness of the proposed technique, the new signal model is applied

for both Zero Forcing and Minimum Mean Square Error detectors. The simulation results revealed that the proposed system outperforms the traditional ZF and MMSE detectors. But, the proposed system performance is highly degraded with the appearance of the grating lobes.

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