

# Design of Flight Control Systems for a Hypersonic Aircraft Using sliding-PID Control

Belkacem Kada\*

*King Abdulaziz University, Department of Aerospace Engineering, P.O. Box 80204, Jeddah, 21589, KSA*

*Email: bkada@kau.edu.sa*

## Abstract

The paper presents the application of sliding-PID control to the design of robust flight control system for a hypersonic aircraft. The proposed controller uses an approach that combines the high-order PID controller with high-order sliding mode (HOSM) control. The PID uses high-order time-derivative (HOTD) function of the sliding mode variable while the HOSM uses the signum function of the HOTD function. HOTD is built using the relative degree nonlinear dynamics of multivariable systems driven by affine control inputs. A displacement autopilot is designed for pitch control of an air-breathing hypersonic vehicle model. Numerical simulation demonstrates the effectiveness of the proposed controller and shows its advantages as compared to the quasi-homogenous HOSM controller.

**Keywords:** Aerospace control systems; hypersonic vehicle; sliding-PID control; sliding mode control.

## 1. Introduction

Flight control systems design for aerospace hypersonic vehicles has been investigated for several decades where a great number of control methods have been developed. Those methods have resulted in different control systems and autopilot topologies with different performance, robustness, and mechanization complexity levels. On the other hand, hypersonic flows and their impact on the aerodynamics and stability of aerospace vehicles impose more challenges for hypersonic vehicles flight control systems (HV-FCS) designers. The early HV-FCS have been developed based on linear dynamic models or linearized nonlinear models around trimmed operating conditions.  $H_\infty$  control [1], linear parameter-varying modeling [2], Reference command tracking [3],  $L_1$  adaptive method [4], linear-quadratic stochastic robust control [5] are among those methods.

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\* Corresponding author.

Linear control methods have shown performance degradation and vulnerability against modeling uncertainties and external disturbances. To overcome the limitations and drawbacks of the linear methods, nonlinear robust and adaptive control has been extensively investigated for aerospace control systems [6-13]. Nevertheless, the effectiveness of these controllers strongly depends on the system model fidelity and the resulting closed-loop control shows mechanization limitations. Sliding mode control (SMC) strategy has been successfully used to design robust control systems for highly nonlinear systems such as hypersonic vehicles with less performance restrictions and implementation complications. However, it is well known that standard SMC suffers from the chattering effect that could harm the controllers and degrade the system performance. High-order sliding mode (HOSM) control has been known as an alternative to the standard SMC for enhanced performance, strong robustness, and free chattering control. Recently, HOSM control has been used under different topologies in the design of HV-FCS [14-20]. In the present paper, the sliding-PID control methodology proposed in [21] is used herein to design high-performance and easy mechanization flight control systems for hypersonic vehicles. A hybrid PID-HOSM controller is constructed based upon the use of the nonlinear system dynamics, local relative degree concept, and discontinuous HOSM control. The methodology is applied to design a displacement autopilot for a hypersonic aircraft to successfully achieve the tracking of desired Angle-Of-Attack (AOA). Sliding-PID tracking performance is compared to a fourth order quasi-continuous HOSM (QC-HOSM) [22,23] where results show the effectiveness of the proposed control method. This paper is organized as follows: In section 2, a nonlinear model of hypersonic vehicle motion is presented. In section 3, the sliding-PID control and the QC-HOSM control are introduced. Section 4 shows the application of sliding-PID control to the design of a displacement autopilot where the results are compared to those provided by a 4th order QC-HOSM control. Conclusions are provided in section 5.

## 2. Hypersonic dynamics modeling

Considering the pitch plane motion of an air-breathing hypersonic vehicle where the vehicle dynamics are given as follows [24].

$$\dot{V} = \frac{T \cos \alpha - D}{m} - \frac{\mu \sin \gamma}{r^2} \quad (1)$$

$$\dot{\alpha} = -\frac{(L + T \sin \alpha)}{mV} + \frac{(\mu - Vr^2) \cos \gamma}{Vr^2} + q \quad (2)$$

$$\dot{q} = \frac{\rho V^2 S \bar{c}}{2I_{yy}} [C_M(\alpha) + C_M(q) + C_M(\delta_e)] \quad (3)$$

$$\dot{\gamma} = q - \dot{\alpha} \quad (4)$$

$$\dot{h} = V \sin \gamma \quad (5)$$

where  $V$ ,  $\alpha$ ,  $\gamma$ ,  $q$ ,  $M$  and  $h$  are the velocity, Angle-Of-Attack (AOA), pitch rate, flight-path angle, Mach number, and altitude of the aircraft, respectively. The aeroloads, pitch moment, and Earth's center are modeled as follows

$$L = \frac{1}{2}\rho V^2 S C_L \quad (6)$$

$$D = \frac{1}{2}\rho V^2 S C_D \quad (7)$$

$$T = \frac{1}{2}\rho V^2 S C_T \quad (8)$$

$$r = h + R_E \quad (9)$$

The aerodynamic coefficients are estimated from wind-tunnel measurements as follows

$$C_L(\alpha) = 0.6203\alpha \quad (10)$$

$$C_D(\alpha) = 0.6450\alpha^2 + 0.0043378\alpha + 0.003772 \quad (11)$$

$$C_T(\xi) = \begin{cases} 0.02576\xi & \text{if } \xi > 0 \\ 0.0224 + 0.003368\xi & \text{if } \xi < 0 \end{cases} \quad (12)$$

$$C_M(\alpha) = -0.035\alpha^2 + 0.036617\alpha + 5.3261 \times 10^{-6} \quad (14)$$

$$C_M(q) = \frac{q\bar{c}}{2V}(-6.796\alpha^2 + 0.3015\alpha - 0.2289) \quad (15)$$

$$C_M(\delta_e) = (0.0292 + \Delta c_e)(\delta_e - \alpha) \quad (16)$$

where  $\delta_e$ ,  $\xi$  are the elevator deflection and throttle setting, respectively. The air density and sound speed are computed, respectively, from the following expressions

$$\rho(h) = 0.00238 \left( \frac{-h}{24,000} \right) \alpha \quad (17)$$

$$a(h) = (8.99 \times 10^{-9} h^2 - 9.16 \times 10^{-4} h + 996) \quad (18)$$

The dynamic model (1)-(5) is a multivariable state-space model

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u} \\ \mathbf{y} &= \mathbf{h}(\mathbf{x}) \end{aligned} \quad (19)$$

where  $\mathbf{x} = [V \ \alpha \ \gamma \ q \ h]^T \in \mathbb{R}^5$  and  $\mathbf{u} = [\beta \ \delta_e]^T \in \mathbb{R}^2$ . The system output can be selected as one of its states such that  $\mathbf{y} = \mathbf{x} \in \mathbb{R}^5$ . The nonlinear mapping  $\mathbf{f}(\mathbf{x}): \mathbb{R}^5 \rightarrow \mathbb{R}^5$  and  $\mathbf{g}(\mathbf{x}): \mathbb{R}^5 \rightarrow \mathbb{R}^5$  are sufficiently smooth

functions

### 3. Hypersonic Sliding-PID and QC-HOSM Control

In this section, both sliding-PID control and quasi-homogeneous HOSM control problems are formulated according to the references [21,23] respectively. The following assumptions are necessary for the development of non-conventional sliding mode control methods [22,25-27].

**A1:** Each state  $x_i (i = 1, \dots, n)$  has a relative degree  $r_i$

**A2:** The control inputs  $u_j (j = 1, \dots, m)$  are supposed bounded

$$\bar{u}_{j,min} \leq u_j \leq \bar{u}_{j,max} \quad (20)$$

**A3:** The vector-valued functions  $\mathbf{f}, \mathbf{f}^{(1)}, \dots, \mathbf{f}^{(r-1)}$  are bounded in Euclidean norm such that

$$\|\mathbf{f}^{(k)}\|_2 \leq \mu_k \quad (21)$$

where  $\mathbf{f}^{(k)}$  is the  $k^{th}$  time-derivative of the vector  $\mathbf{f}$ .

**A4:** If the relative degree of an output  $y$  is  $r$ ,  $y^{(r)}$  is written as follows

$$y^{(r)} = \phi(\mathbf{x}, t) + \psi(\mathbf{x}, t)u \quad (22)$$

with

$$\begin{cases} \phi(\mathbf{x}) = L_f^r y(\mathbf{x}) = 0 \\ \psi(\mathbf{x}) = L_g L_f^{r-1} y(\mathbf{x}) \neq 0 \end{cases} \quad (23)$$

where  $L_f^r, L_f^{r-1}$ , and  $L_g$  are the Lie derivatives.

**A5:** The tracked (desired) output  $y_d(t)$  (the subscript 'd' denotes the desired value) has the same relative degree  $r$  as the tracking output  $y(t)$  and,

$$|y_d^{(k)}(t)| \leq \xi_k \quad (k = 0, 1, \dots, r) \quad (24)$$

#### 3.1. QC-HOSM control

For an uncertain multivariable system with  $\mathbf{x} \in \mathbb{R}^n$ ,  $\mathbf{u} \in \mathbb{R}^m$ , and  $\mathbf{y} \in \mathbb{R}^p$  (n states, m inputs, and p outputs), the HOSM control concept is equivalent to force the state trajectories of such system to evolve or move on the following integral  $r$ -order set of sliding manifolds

$$\sigma^r = \left\{ \mathbf{x} \in R^n \left| \begin{array}{l} \sigma_1 = \dot{\sigma}_1 = \dots = \sigma_1^{(r_1-1)} = 0 \\ \vdots \\ \sigma_p = \dot{\sigma}_p = \dots = \sigma_p^{(r_p-1)} = 0 \end{array} \right. \right\} \neq \emptyset \quad (25)$$

where  $\mathbf{r} = [r_1 \ r_2 \ \dots \ r_p]^T$  denotes the sliding order vector with respect to the output vector  $\mathbf{y}$ , and  $\sigma(\mathbf{x}) = \mathbf{h}(\mathbf{x}) - \mathbf{y}_d(t)$  is the new output constraint vector of the system.

The relative degree  $r_i$  characterizes the dynamics smoothness degree in the vicinity of the  $r_i$ -sliding mode, hence  $r_i$  should be known and constant. This statement is equivalent to the condition stated in assumption A5. It results that

$$\left[ L_f^{r_1} \sigma_1(\mathbf{x}) \ \dots \ L_f^{r_p} \sigma_p(\mathbf{x}) \right]^T = \mathbf{A}(\mathbf{x}) + \mathbf{B}(\mathbf{x})\mathbf{u}_r \quad (26)$$

with

$$\mathbf{A}(\mathbf{x}) = \left[ L_f^{r_1-1} \sigma_1(\mathbf{x}_0) \ \dots \ L_f^{r_p-1} \sigma_p(\mathbf{x}_0) \right]^T \in R^p \quad (27)$$

$$\mathbf{B}(\mathbf{x}) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} \sigma_1(\mathbf{x}_0) & \dots & L_{g_m} L_f^{r_1-1} \sigma_1(\mathbf{x}_0) \\ \vdots & & \vdots \\ L_{g_1} L_f^{r_p-1} \sigma_p(\mathbf{x}_0) & \dots & L_{g_m} L_f^{r_p-1} \sigma_p(\mathbf{x}_0) \end{bmatrix} \in R^{p \times m} \quad (28)$$

The asymptotic solution to the system (25) is guaranteed if and only if  $\mathbf{B}(\mathbf{x})$  is nonsingular. For the case  $p > m$  where  $\mathbf{B}(\mathbf{x})$  is non-square, the solution to the system (25) requires that the following matrix is nonsingular

$$\mathbf{B}^l = \mathbf{B}^T (\mathbf{B}\mathbf{B}^T)^{-1} \quad (29)$$

It results that the objective of HOSM is to design controllers  $u_j(\mathbf{x})$  to enforce the sliding variable  $\sigma_i(\mathbf{x})$  associated to the output  $y_i$  to reach their zero-level in finite time in despite of disturbances and uncertainties and without chattering of controllers.

### 3.2. Sliding-PID control

Suppose that the assumptions A1 to A5 hold, using the relative degree  $r$  of a single-input-single-output  $u \rightarrow y$  subdynamics of the system (19), a sliding-PID controller can be designed as follows [21]

$$u_r(x) = u_{PID} + u_s = K_p \sigma(x) + K_d \dot{\sigma}(x) + K_i \int_{t_0}^t \sigma(x, \tau) d\tau - K_s \text{sign} \left( \varphi(\sigma, \dot{\sigma}, \dots, \sigma^{(r-1)}) \right) \quad (30)$$

where

$$\varphi(\sigma, \dot{\sigma}, \dots, \sigma^{(r-1)}) = \sum_{k=r-1}^0 \lambda_k L_f^k \sigma(x, t) \tag{31}$$

The sliding-PID controller (30)-(31) is implemented as shown in Figure 1.

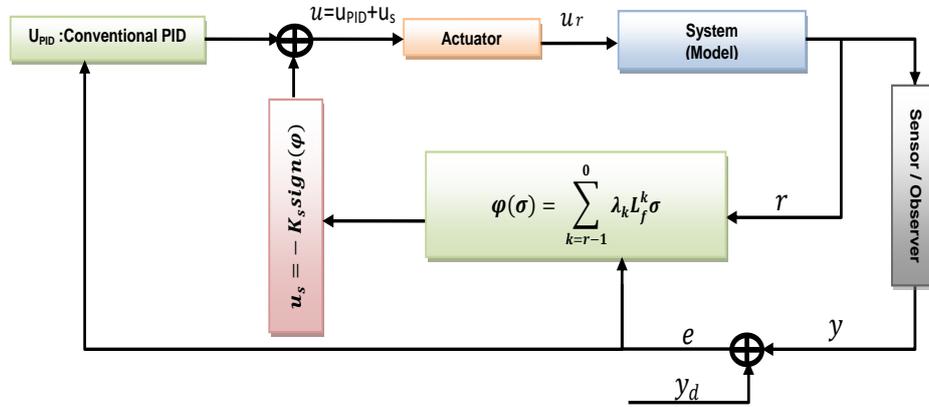


Figure 1: Block diagram of the sliding-PID controller

### 3.3. Quasi-homogeneous HOSM (QC-HOSM) control

Under the assumptions that the terms  $A_i(x)$  and  $B_{ij}(x)$  are some bounded uncertain smooth functions and that the control inputs  $u_j(x)$  are some Lebesgue-measurable bounded signals, the problem described by (26) is standard and could be solved by the following set of known  $r_i$ -sliding controllers (this statement is an extension of the statement stated in for a Single-Input-Single-Output system)

$$\mathbf{u}_r(\mathbf{x}) = \left\{ \begin{array}{c} -G_1 \varphi_{r_1}(\sigma_1, \dot{\sigma}_1, \dots, \sigma_1^{(r_1-1)}) \\ \vdots \\ -G_p \varphi_{r_p}(\sigma_p, \dot{\sigma}_p, \dots, \sigma_p^{(r_p-1)}) \end{array} \right\} \tag{32}$$

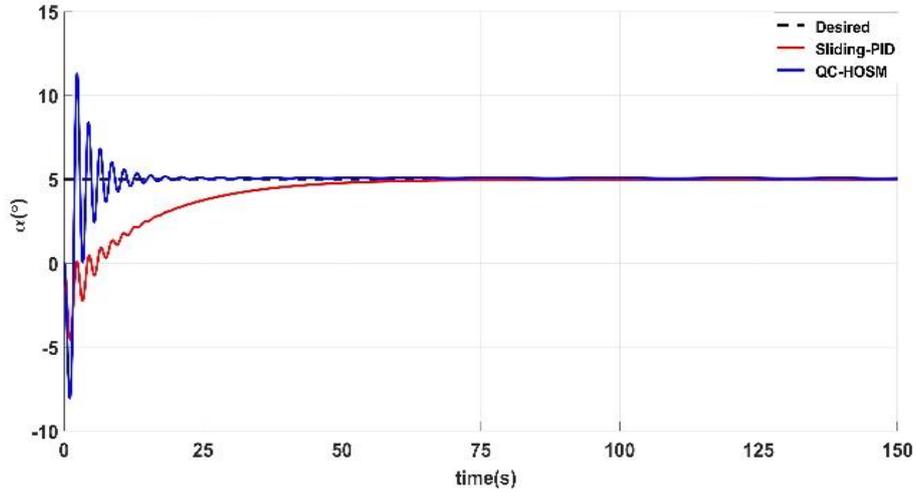
The gains  $G_i$  are adjustable gains introduced to compensate model uncertainties, unmodeled dynamics and external disturbances. The control laws  $\Psi_{r_i}(\sigma_i, \dot{\sigma}_i, \dots, \sigma_i^{(r_i-1)})$  can be selected to be the  $r$ -sliding functions. The corresponding quasi-continuous controllers are constructed as shown in the following procedure

$$\varphi_{r_i}(\sigma) = -G_i \frac{\Phi_{r_i-1, r_i}}{N_{r_i-1, r_i}} \tag{33}$$

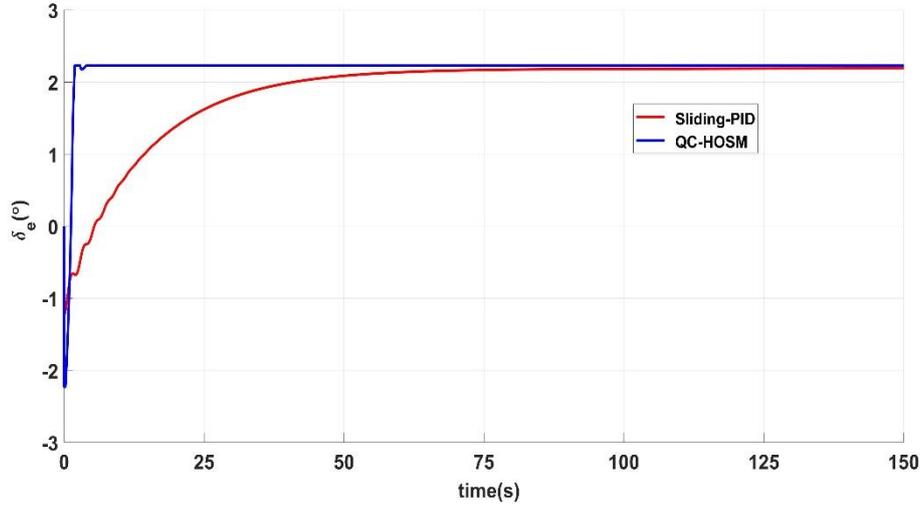


**Table 1:** Controllers gains for the AOA tracking

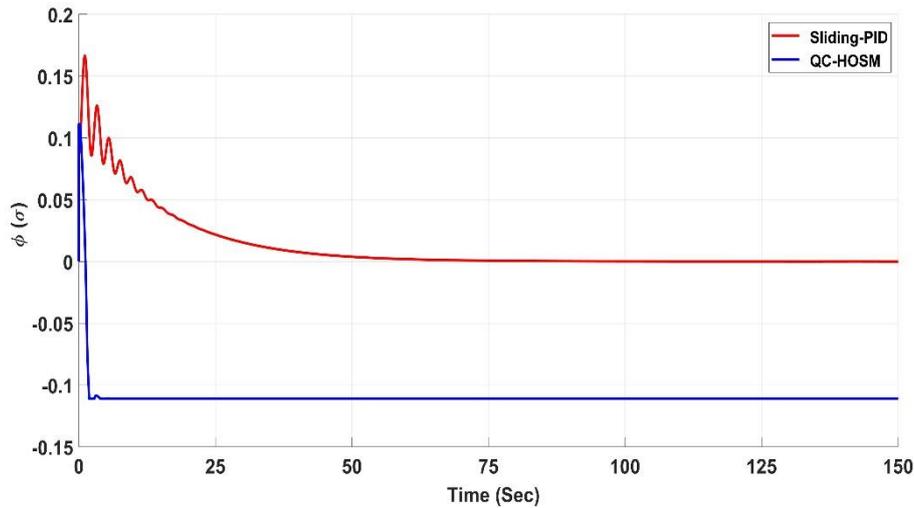
Controller	Gains
Sliding-PID	$K_p = 0.51, K_d = 1, K_i = 0.35, \lambda_0 = 1, \lambda_1 = 1; K_s = -0.26$
QC-HOSM	$\beta_1 = 5, \beta_2 = 2, \beta_3 = 2; G = 0.35$
HOSM observer	$\lambda_0 = 7, \lambda_1 = 10, \lambda_2 = 5, \lambda_3 = 3, L = 0.01$



**Figure 2:** Time-history response of AOA tracking



**Figure 3:** Control effort



**Figure 4:** Convergence of the function  $\phi(\sigma)$

From the results above, it can be seen that the proposed strategy offer a good solution to the design of automatic control systems for hypersonic aerospace vehicle. The control scheme provides fast response with smooth control input, which is often desired in control practices.

## 5. Conclusion

In this paper, a displacement autopilot for hypersonic vehicle is designed using sliding-PID control method that combines the conventional PID with discontinuous-HOSM control. As application, an AOA autopilot was designed and simulated for track and maintain a desired output. Simulation results showed good performance tracking without control input chattering. The results also revealed that the sliding-PID outperformed the quasi-continuous HOSM. As constraints of the proposed control scheme, we can mention that the computation of time derivatives requires, in addition to the robust differentiator (HOSM-differentiator), a signal filtering to clear the signals from the disturbance and measurement noises.

## 6. Recommendations

As future work, the control method will be extended to the design of other types of autopilots for hypersonic automatic flight control systems. Also the trail-error tuning of the control gains can be done by an adaptive process using adaptive laws

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