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Dynamical Behavior of Brusselator System Driven by Non-Gaussian Noise

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Abstract

The non-Gaussian noise induced the mean first passage time (MFPT) in Brusselator system are examined. In this paper, the path integral method is used to approximate non-Gaussian noise to Gaussian color noise. The FPT of the 50000 response tracks is obtained by solving the system equation through the fourth-order stochastic Runge-Kutta algorithm. Then we get the MFPT. The influences of the noise intensity, correlation time and non-Gaussian parameter of non-Gaussian noise on the MFPT are characterized. We also found the noise enhanced stability (NES) phenomenon in the system.

Keywords: Non-Gaussian noise; Brusselator model; Mean first passage time; Noise enhanced stability.

1. Introduction

In many nonlinear systems, it is common to add noise to the equation to study the fluctuations. The research on the influence of noise shows that in many cases, noise does not have a bad impact on the system, on the contrary, noise will have a constructive effect on the system. In recent years, the study of nonlinear dynamics with external noise sources has led to the discovery of some similar resonance phenomena, such as, stochastic resonance (SR) [1-4], resonant activation (RA) [5], and noise enhanced stability (NES) [6]. All these phenomena are characterized by non monotonic behavior as a function of noise intensity or parameters of noise, which reflects the constructive influence of noise on nonlinear systems. The MFPT is to study the mechanism of the mutual transformation between system states. In recent years, MFPT has attracted wide attention of scholars in various fields.

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Breen and his colleagues [7] characterized an arbitrary directed matrix reaching an equal directed graph on the lower bound of the maximum mean first passage time, thus generating a kind of Markov chain with optimal short-term behavior. Fiasconaro and his colleagues [8] explored the MFPT of Brownian particle from an initial unstable state in metastable underdamped system and found the typical NES effect by MFPT with a visible hump structure or a divergent behavior. Deng and his colleagues [9] used the Brusselator model to study the structure of the construction energy service industry system, and established an equation based on the entropy method to verify whether the construction energy service industry system has dissipative structure. According to the research results, it provides a scientific basis for policy making. Li and his colleagues [10] studied the mean first passage time of a piecewise nonlinear model driven by color correlated noise. Therefore, this paper focuses on the dynamic behavior of the Brusselator system driven by non-Gaussian noise.

2. The Brusselator model and non-Gaussian noise

Brusselator system is an autocatalytic chemical oscillation model. It was proposed by Prigogine and Lefever [11].

The free Brusselator model is as follows:

$$\begin{aligned} \frac{dx}{dt} &= a-(1+b)x+x^2 y, \\ \frac{dy}{dt} &= bx-x^2 y, \end{aligned} \tag{1}$$

According to the reference [12], the point $\left(x_s = a, y_s = \frac{b}{a}\right)$ is the equilibrium point of deterministic model (1), and it loses its stability at the supercritical Hopf bifurcation $b_{hp} = 1+a^2$ and oscillations happen for $b > b_{hp}$. The Langevin equation corresponding to the Brusselator model is:

$$\begin{aligned} \frac{dx}{dt} &= a-(1+b)x+x^2 y, \\ \frac{dy}{dt} &= bx-x^2 y+\eta(t), \end{aligned} \tag{2}$$

The $\eta(t)$ donates non-Gaussian noise. The non-Gaussian noise $\eta(t)$ satisfies the following Langevin equation[13,14]:

$$\frac{\mathrm{d}\eta(t)}{\mathrm{dt}} = -\frac{1}{\tau}\frac{\mathrm{d}}{\mathrm{d\eta}}V_q(\eta)+\frac{1}{\tau}\varepsilon(t), \tag{3}$$

Where τ is the correlation time of non-Gaussian noise $\eta(t)$, and $V_q(\eta)$ satisfies

$$V_q(\eta) = \frac{D}{\tau(q-1)} \ln\left[1 + \frac{\tau}{D}(q-1)\frac{\eta^2}{2}\right]$$

$\varepsilon(t)$ is Gaussian white noise, and $\varepsilon(t)$ satisfies

$$\begin{aligned} &\langle \varepsilon(t) = 0 \rangle \\ &\left\langle \varepsilon(t)\varepsilon(t^{'}) \right\rangle = 2D\delta(t - t^{'}) \end{aligned} \tag{4}$$

Where D is the noise intensity of Gaussian white noise $\varepsilon(t)$, and δ is delta function.

The statistical properties of $\eta(t)$ are as follows:

$$\begin{aligned} &\langle \eta(t) \rangle = 0 \\ &\langle \eta^2(t) \rangle = \begin{cases} \dfrac{2D}{\tau(5-3q)}, q \in \left(-\infty, \dfrac{5}{3}\right) \\ \infty, q \in \left[\dfrac{5}{3}, 3\right) \end{cases} \end{aligned} \tag{5}$$

When $|q-1| << 1$, the non-Gaussian noise $\eta(t)$ can be expressed as follows by using path integral method[13,14]:

$$\frac{\mathrm{d}\eta(t)}{\mathrm{dt}} = -\frac{1}{\tau_{\mathrm{eff}}}\eta(t) + \frac{1}{\tau_{\mathrm{eff}}}\varepsilon_1(t), \tag{6}$$

Here is a Gaussian white noise which statistical characteristics can be represented by their mean and variance,

$$\begin{aligned} &\langle \varepsilon_1(t) \rangle = 0, \\ &\left\langle \varepsilon_1(t)\varepsilon_1(t^{'}) \right\rangle = 2D_{\mathrm{eff}}\delta(t - t^{'}). \end{aligned} \tag{7}$$

Where τ_{eff} and D_{eff} are the effective noise correlation time and the effective noise intensity, respectively.

$$\begin{aligned} \tau_{\mathrm{eff}} &= \frac{2(2-q)}{5-3q}\tau, \\ D_{\mathrm{eff}} &= \left(\frac{2(2-q)}{5-3q}\right)^2 D. \end{aligned} \tag{8}$$

The non-Gaussian parameter q indicates the degree of deviation of $\eta(t)$ from the Gaussian distribution. When $q \to 1$, $\eta(t)$ can be approximately regarded as a Gaussian colored noise with correlation time τ_{eff} and noise intensity D_{eff}.

3. The MFPT of The Brusselator System

We take a=1,b=1,996, and the following analysis is based on these parameters.

3.1 Simulation methods

Non-Gaussian noise has non Markov property. Because of the existence of non-Gaussian noise in the system, the theoretical method is difficult to solve the equation, so we consider the numerical simulation method to solve the equation (2).

We use the fourth-order Runge-Kutta algorithm to simulate the system (2):

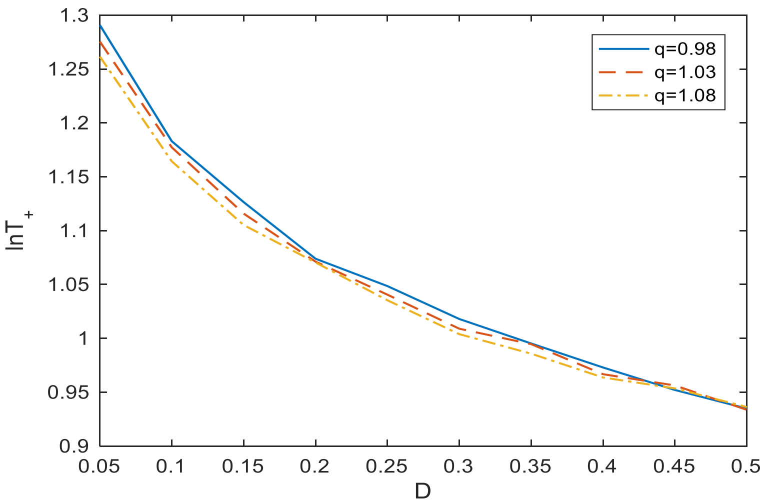
$$\begin{cases} x_{i+1} = x_i + \Delta t/6\left(k_1 + 2k_2 + 2k_3 + k_4\right) \\ y_{i+1} = y_i + \Delta t/6\left(l_1 + 2l_2 + 2l_3 + l_4\right) \\ z_{i+1} = z_i + \Delta t/6\left(m_1 + 2m_2 + 2m_3 + m_4\right) + \dfrac{s_2\sqrt{2D\Delta t}}{\tau} \end{cases} \tag{9}$$

where,

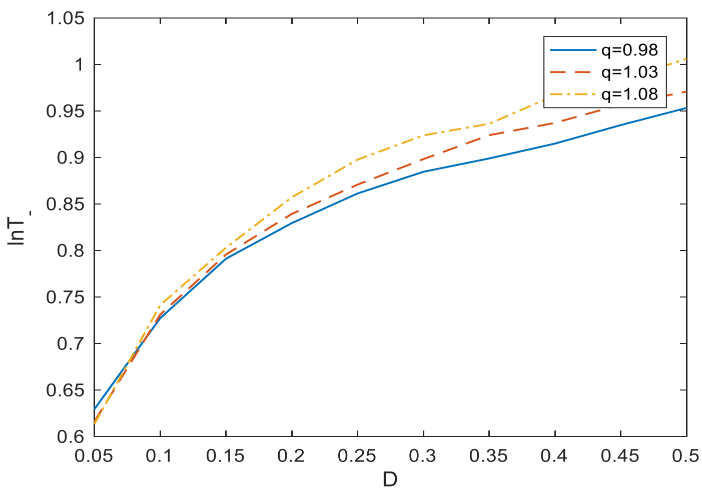
$$k_1 = a - (1+b)x_i + x_i^2 y_i$$
$$k_2 = a - (1+b)(x_i + \Delta t/2 \cdot k_1) + (x_i + \Delta t/2 \cdot k_1)^2 (y_i + \Delta t/2 \cdot l_1)$$
$$k_3 = a - (1+b)(x_i + \Delta t/2 \cdot k_2) + (x_i + \Delta t/2 \cdot k_2)^2 (y_i + \Delta t/2 \cdot l_2)$$
$$k_4 = a - (1+b)(x_i + \Delta t \cdot k_3) + (x_i + \Delta t \cdot k_3)^2 (y_i + \Delta t \cdot l_3)$$
$$l_1 = bx_i - x_i^2 y_i + z_i$$
$$l_2 = b(x_i + \Delta t/2 \cdot k_1) - (x_i + \Delta t/2 \cdot k_1)^2 (y_i + \Delta t/2 \cdot l_1) + z_i + \Delta t/2 \cdot m_1 + s_1\sqrt{2D\Delta t}$$
$$l_3 = b(x_i + \Delta t/2 \cdot k_2) - (x_i + \Delta t/2 \cdot k_2)^2 (y_i + \Delta t/2 \cdot l_2) + z_i + \Delta t/2 \cdot m_2 + s_1\sqrt{2D\Delta t}$$
$$l_4 = b(x_i + \Delta t \cdot k_3) - (x_i + \Delta t \cdot k_3)^2 (y_i + \Delta t \cdot l_3) + z_i + \Delta t \cdot m_3 + s_1\sqrt{2D\Delta t}$$
$$m_1 = -\frac{1}{\tau} \cdot \frac{z_i}{1 + \frac{\tau}{D}(q-1) \cdot \frac{z_i^2}{2}}$$
$$m_2 = -\frac{1}{\tau} \cdot \frac{z_i + \Delta t/2 \cdot m_1 + s_2\sqrt{2D\Delta t}}{1 + \frac{\tau}{D}(q-1) \cdot \frac{(z_i + \Delta t/2 \cdot m_1 + s_2\sqrt{2D\Delta t})^2}{2}}$$
$$m_3 = -\frac{1}{\tau} \cdot \frac{z_i + \Delta t/2 \cdot m_2 + s_2\sqrt{2D\Delta t}}{1 + \frac{\tau}{D}(q-1) \cdot \frac{(z_i + \Delta t/2 \cdot m_2 + s_2\sqrt{2D\Delta t})^2}{2}}$$
$$m_4 = -\frac{1}{\tau} \cdot \frac{z_i + \Delta t \cdot m_3 + s_2\sqrt{2D\Delta t}}{1 + \frac{\tau}{D}(q-1) \cdot \frac{(z_i + \Delta t \cdot m_3 + s_2\sqrt{2D\Delta t})^2}{2}}$$

So we can get the numerical solution of the Brusselator system driven by non-Gaussian noise. For nonlinear dynamic system, its dynamic properties include steady-state and transient properties. The transient properties can be described by escape rate or MFPT. The MFPT is to study the mechanism of the mutual transformation between system states. Here we choose two states of the Brusselator system, one is stable state: v_1： (1,1.996), and the other is unstable state: v_2： (0.5,1.996). In Brusselator system, the FPT of particles in two directions is different, so this paper studies the two directions ($v_1 \to v_2$ and $v_2 \to v_1$) respectively. In the numerical simulation, the initial value $v = v_1$ (or $v = v_2$) is given firstly, and the time series of the system response is obtained according to the formulas (9). The time required for the particle to enter state v_1 (or v_2) from state v_2 (or v_1) for the first time is recorded. Based on this method, the time of 50000 the response series are obtained.

3.2 The effects of parameters on MFPT



(a) $\ln T_+ (v_1 \to v_2)$

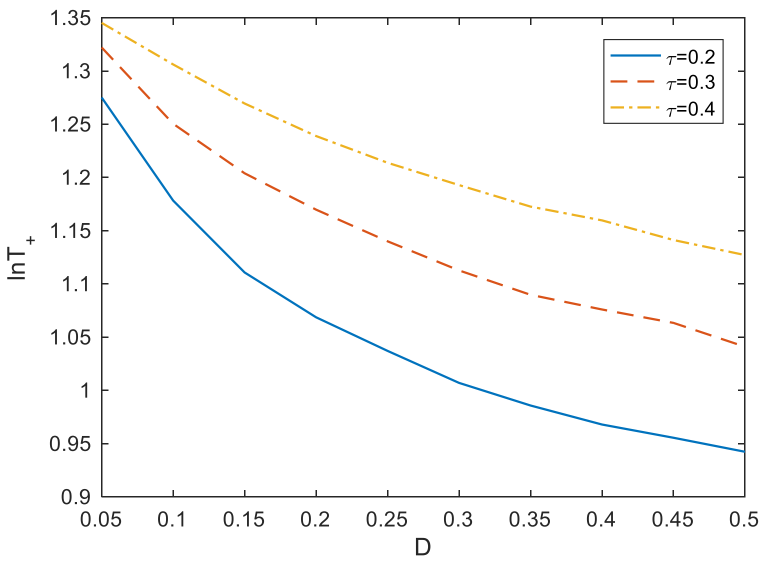


(b) $\ln T_- (v_2 \to v_1)$

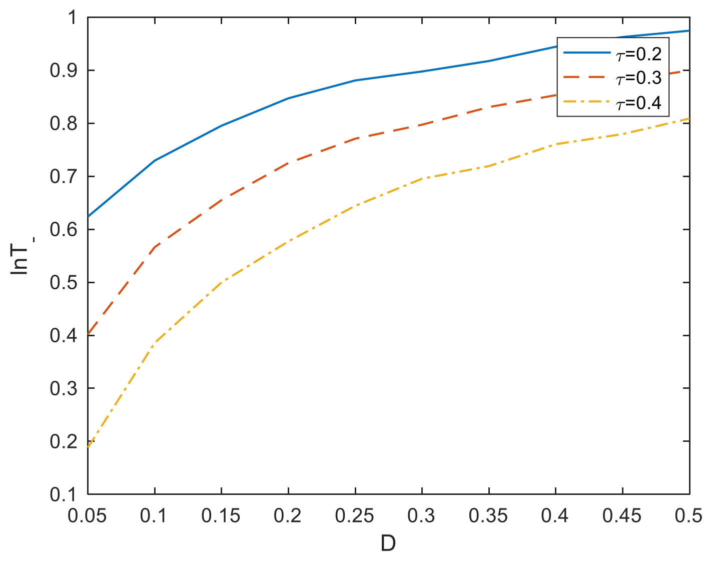
Figure 1: MFPT as a function of noise intensity D for different values of q ($\tau = 0.2$).

According to the numerical simulation methods of section 3.1, the influence of non-Gaussian parameter q, correlation time τ and additive noise intensity D on MFPT in two directions $T_+(v_1 \rightarrow v_2)$ and $T_-(v_2 \rightarrow v_1)$ is discussed.

In Figure 1(a), we depict the results of $\ln T_+$ as a function of noise intensity D with different values of non-Gaussian parameter q. From the image, we can see that $\ln T_+$ gradually decreases as D increases. Similarly, $\ln T_+$ gradually decreases as q increases. It is shown that the larger additive noise intensity D and the larger non-Gaussian parameter q are beneficial to the transition of the concentration of intermediate substances from steady state to unsteady state and the production of products. Figure 1(b) shows $\ln T_-$ as the function of D for different q. It can be seen from the figure that the time required for crossing from $v_2 \rightarrow v_1$ increases gradually with the increase of D. For smaller D, when D is fixed, $\ln T_-$ decreases with the increase of q. However, for larger D, when D is fixed, $\ln T_-$ increases with the increase of q .The results show that when the additive noise intensity D is small, decreasing the value of non-Gaussian parameter q inhibits the transition of the concentration of intermediate substances from steady state to unsteady state, which is conducive to the formation of products. When the additive noise intensity D is large, the same effect can be achieved by increasing non-Gaussian parameters q.



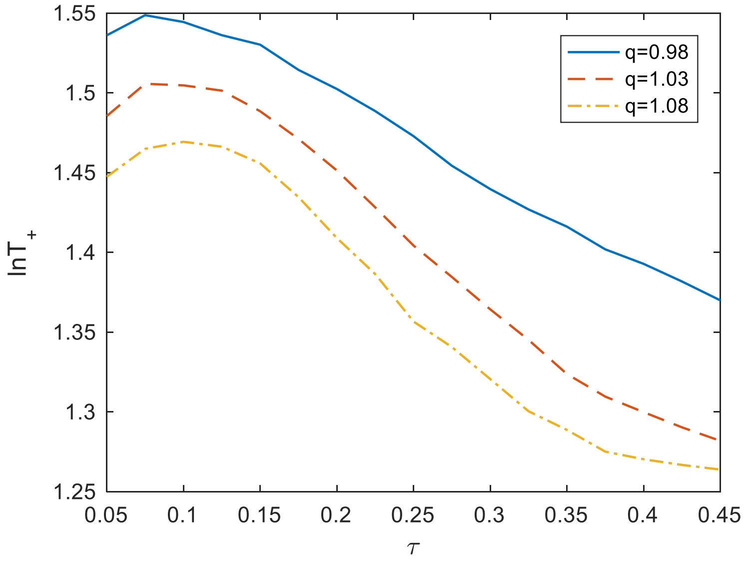
(a) $\ln T_+(v_1 \rightarrow v_2)$



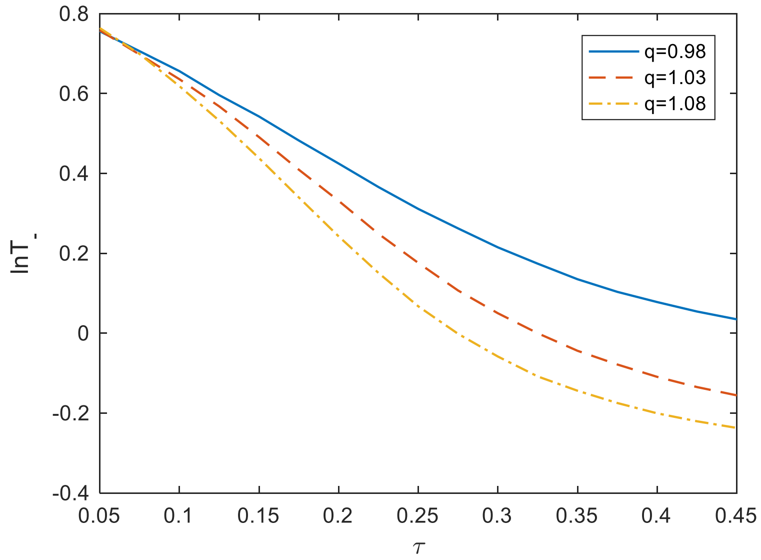
(b) $\ln T_{-}(v_2 \to v_1)$

Figure 2: MFPT as a function of noise intensity D for different values of τ ($q=1.03$).

In Figure 2(a), $\ln T_+$ is presented as a function of the noise intensity D for different values of τ. We can find that the time required for crossing from $v_1 \to v_2$ decreases gradually with the increase of D. For fixed D, $\ln T_+$ increases as the τ increases. It is shown that the larger additive noise intensity D and the smaller correlation time τ are beneficial to the transition of the concentration of intermediate substances from steady state to unsteady state and the production of products. Figure 2(b) shows $\ln T_-$ as the function of D for different τ. From the image, we can get that the time required for crossing from $v_2 \to v_1$ increases gradually with the increase of D. For fixed D, $\ln T_-$ decreases as the τ increases. The outcomes imply that the decrease of correlation time τ and the increase of additive noise intensity D inhibit the transition of the concentration of intermediate substances from unsteady state to steady state, and indirectly promote the production of products.



(a) $\ln T_{+}(v_1 \to v_2)$



(b) $\ln T_{-}(v_2 \to v_1)$

Figure 3: MFPT as a function of correlation time τ for different values of q ($D = 0.01$).

In Figure 3(a), $\ln T_{+}$ as a function of correlation time τ for different values of q. From the image, we can see that MFPT shows a non-monotonic dependence with the increase of correlation time τ. We can see that there exists a critical value. When τ is on the left of the critical value, $\ln T_{+}$ increases gradually with the increase of τ, and the curve shows an upward trend. As τ continues to increase, the value of $\ln T_{+}$ begins to decrease and the curve begins to decline. At the same time, there exists a maximum in the image. At this maximum, the transition speed of particles is the slowest. This means that the τ value at this time can inhibit the transition of intermediate substances from steady state to unsteady state, and make the system in a state independent of external equilibrium. We think this behavior is called NES effect. When the correlation time τ is fixed, the larger non-Gaussian parameter q is beneficial to the transition of the system. Figure 3(b) shows $\ln T_{-}$ as a function of correlation time τ under different non-Gaussian parameter q. It can be seen from the figure that the smaller correlation time τ and the smaller non-Gaussian parameter q inhibit the transition of the concentration of intermediate products from unsteady state to steady state, thus indirectly promoting the production of products.

4. Conclusions

The purpose of this paper is to discuss the dynamical behavior of Brusselator system driven by non-Gaussian noise. Firstly, the non-Gaussian noise is approximated to Gaussian colored noise by path integral method. The fourth-order stochastic Runge Kutta algorithm is used to solve the system equation. Then we get the 50000 response tracks. Then we get the MFPT. Based on the numerical solution, the effects of the additive non-Gaussian noise on the MFPT are discussed. The results imply that non-Gaussian noise can induce the phenomenon of NES. These phenomena are similar to those in reference [8]. The influence of additive non-Gaussian noise intensity D on the system transition is sophisticated in different directions. With the increase of additive noise intensity, the transition time of particles on v_1 to v_2 decreases. The results show that the increase of D could be conducive to the formation of reactants. In the other direction, the result is exactly the opposite. The influence of correlation time τ on the transition of particles is different. In the direction of v_1 to v_2, we can see clearly that MFPT shows a non monotonic dependence with the change of correlation time τ. This indicates that noise can induce NES. In the direction of v_2 to v_1, with the increase of correlation time τ, the transition time of particles in this direction decreases. The results demonstrate that the increase of τ is not conducive to the formation of reactants. In addition, the influence of non-Gaussian parameter on particle transition in two directions is also sophisticated. In the direction of v_1 to v_2, particle transition time is reduced as q grows. The results reflect that the increase of q is beneficial to the formation of reactants. In the other direction, the influence of non-Gaussian parameter q on the transition of particles is different, and the influence mode is influenced by D. When D is small, the transition speed increases with the increase of q. When D is large, the transition speed decreases with the increase of q.

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