

# Probabilistic Load Flow by Monte Carlo Simulation to Determine the Amount of Use of the Transmission System

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## Abstract

Access to the transmission network by generators and distributors is essential in the unbundled configuration of the electricity market. Regulatory agents seek to improve the relationship between network users and their operators in order to optimize investments, increase reliability and lower tariffs. In this context, the need arises to foresee the use of transmission networks as accurately as possible to correctly signal to planners and to reduce users' tariff costs. This paper aims to present a methodology for obtaining the amounts of power at the connection points between users and the transmission network, in addition to a search system for the least probable charge. A probabilistic load flow method by numerical method was used, with the search for the lowest charge value. The inputs were modeled as random variables with known probability distribution functions. High concentrations of wind generators were considered and modeled as Weibull distributions. A modified IEEE 14 bus system was used as a case study. The results showed the amounts by risk and by lower value, being the user's option to choose each of the results. The presented method proved to be an interesting proposal for situations that require a forecast of the amounts of use of the transmission system in the medium and long term, with the presence of wind generators.

**Keywords:** Electricity Distribution System; Probabilistic Load Flow; Monte Carlo Simulation; Amount of Use of the Transmission System.

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## **1. Introduction**

Since the 1990s, the restructuring of the electricity sector has resulted in a wide variety of market models with shifting emphasis in different areas of the structure of the electricity sector and in each country. Traditionally organized vertically and followed by geographic monopolies, the attempt to introduce competition wherever possible became commonplace in the electricity sector. The electrical systems were largely divided into three distinct sectors: generation, transmission, and distribution. In some cases, with a fourth in the form of commercialization sector [1-2] with dispatch and operation were also being considered a fifth division [3]. In this sense, competition was used in the generation and commercialization. Transmission and distribution, on the other hand, were characterized as natural monopolies, and to allow fair competition in other sectors, they needed to be regulated in order to guarantee the functioning and expansion of the system, and competitive prices [1]. Interactions between transmission system and other agents were followed by five main points: conditions of access, pricing, expansion and updating of the system, methods of contracting transmission and ownership and control arrangements for transmission [2]. In all cases, studies have been carried out to improve the usage of distribution networks by market agents. For example, in much of the European Union, transmission contracts are long-term so companies and operators can manage risk and plan for its expansion [4]. In Brazil, electricity distributors must inform the national regulatory agent the maximum load at points of connection with the transmission network for the next four years [5]. Thus, there is a great demand for understanding the dynamics and uncertainties involved in the interface between the transmission and other agents connected to it. Deterministic load flow (DLF) is commonly used for system planning and operation; however, the solution depends heavily on the accuracy of the input data [6]. To consider uncertainties, probabilistic methods are more appropriate [7]. References [7,8] presented extensive reviews on the bases and applications of probabilistic load flow (PLF) in its different mathematical bases and computational techniques. The Brazilian case is also studied with the use of PLF in [9-11], which in addition to PLF, network and flow optimization methods were also applied, along with searching for optimal cost. Thus, the objective of the present paper is to present an application of PLF method to the problems of planning and access to the transmission network, considering the uncertainties both in loads and in electrical network, using the Brazilian regulation framework. To demonstrate the application, loads and generations connected from the IEEE 14 bus system were modeled, using appropriate probability distributions for each type of application. The modified IEEE 14 bus system will represent an interface zone between a distribution system with two connection points with the transmission network. The proposed PLF problem was solved from a numerical approach, e.g., Monte Carlo, simulating DLF many times, hence seeking an approximation of the analytical solution for probabilistically determined input data. Finally, a search for the best solution was carried out using the joint probability of the points of interest. This paper is organized in 4 sections plus this introduction. In section 2 the problem is characterized with a brief review of its most relevant aspects. Section 3 presents the methodology used with presentation of examples. In turn, the fourth section has a dual purpose as a numerical case, with results and discussion. Finally, the conclusion is presented in Section 5.

## **2. Problem characterization**

Changes resulting from restructuring electrical sector had a profound impact on planning and operation of the

distributors, as well as other agents in the sector. The need for agreements between operators, owners and users of the transmission system is essential for planning the line's operation and expansion [2]. In this sense, and in most countries, as transmission networks are a natural monopoly, transmission agents cannot discriminate over access to their lines [1]. However, it is necessary to maintain a balance between installed capacity, short-term demand, and long-term expansion planning. For this reason, long-term contracts for use of transmission systems are practically the rule in most countries [4]. Brazil, which has an integrated transmission network, called the National Interconnected System (NIS), improves special attention to planning and operating of its network. National regulatory agency of Brazilian's electrical system through Normative Resolution No. 666 of 2015 requires generators and distributors to inform the system operator the expected usage amounts, e.g. the maximum monthly loads, for the connection points between distributors or generators with the grid transmission. Expected amounts must be reported for the following four years, with penalties on usage charges in the event of non-compliance with the reported value beyond a tolerance range [5]. That set of information provided by all agents connected to the network allows the system operator and transmission agents to plan the medium- and long-term expansion and operation of the system. In terms of structure, the planning allows reinforcement or construction of new lines to be selected based on more realistic scenarios. On the tariff side, better planning avoids the over-dimensioning of lines that lead to investment costs for consumers [12].

### **2.1. Uncertainties**

Basically the uncertainties in the planning and operation of electrical systems originates from two sources: (i) environmental and social factors and (ii) demographic and economic factors [8]. The first is relevant for operational studies and the second is essential for expansion planning. Application of PLF allows to transform these uncertainties into random variables, defined in terms of probability density functions (PDF), to outputs that are also random variables defined by PDF [13]. Uncertainties must be probabilistically modeled to be used in PLF simulations in that context. For the proposed problem, in general, uncertainties can be grouped into two sets: (i) uncertainties related to input data related to generation and load; (ii) uncertainties related to network, its components and parameters [7,8]. The uncertainties associated with generation are strongly associated with the type of dispatch and energy source of the generators connected to the measurement point. The large presence of intermittent renewable energy sources such as solar photovoltaics and wind is usually described as a continuous distribution [8]. Many studies have shown that wind speed and the load distribution generated at points with a high concentration of wind generators can be modeled as a two-parameter Weibull distribution [7,14,15]. The other traditional sources of energy, on the other hand, are more often modeled as distributions of the Gaussian type, e.g., Normal type. In turn, uncertainties associated with loads connected to the network, when they do not have a profile that can be associated with some known distribution, are modeled after a Gaussian distribution, with the average value chosen in a deterministic way and a standard deviation between +/-5% or +/- 10% from average [8]. Uncertainties associated with the network structure are concentrated in two types: (i) uncertainties in topology and (ii) uncertainties in parameters. Inevitably, changes in network configurations or topology lead to changes in the load flow and on outputs from simulations performed. One of the ways to model topology uncertainties is to consider the network configuration as a discrete stochastic variable with a specific probability distribution for each component, with the final result being the sum of the specific weights of each component across the system [7]. Another random phenomenon related to the transmission networks is the variation of

network parameters due to temperature variation. Usually in PLF studies, the temperature at which conductors and equipment are subjected is considered to be known and constant [7,8].

## 2.2. PLF assessment techniques

PLF can be approached using both numerical and analytical methods. A review of variations within the analytical and numerical methods is presented in [8]. The central idea from analytical approach is to employ concepts of arithmetic, i.e., using convolution techniques, with the Probability Density Functions (PDF) of the random variables of load input data to obtain the PDF of the system states and load flow in the lines in form of mathematical expressions. However, the solution of the PLF by convolution methods is more complex mainly by two points [7]: load flow equations are non-linear; load variables of the different buses are not completely independent, or they are linearly correlated. Thus, many premises need to be considered to execute the PLF. The numerical approach, e.g., Monte Carlo method (MCM) is a method generally used for complex and non-linear systems associated with random variables. This method is based on the repetitive and random substitution of the stochastic variables values and system parameters, in a random sample form, and then execution of deterministic analysis, without the need to simplify the non-linear load flow equations. The MCM is even used as a reference when comparing other methods of PLF because of its precision [13]. However, despite its high precision, computational time can be high, due to the large number of calculations necessary for determination of non-linear power flows [7], which can make its use unattractive in situations that require high speed in obtaining the results. Two main characteristics of MCM are the generation of random numbers and random sampling. Although sampling techniques can be quite sophisticated, PLF through MCC is, in principle, the repetition of a large number of times from DLF with the input data in different random combinations within determined distributions. Thus, it is possible to apply exactly the non-linear equations presented from (1) to (5) [16]:

$$P_i = V_i \sum_{k=1}^n V_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \quad (1)$$

$$Q_i = V_i \sum_{k=1}^n V_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) \quad (2)$$

$$P_{ik} = -t_{ik} G_{ik} V_i^2 + V_i V_k (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik}) \quad (3)$$

$$Q_{ik} = t_{ik} B_{ik} V_i^2 - B_{ik} V_i^2 + V_i V_k (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik}) \quad (4)$$

$$Q_{i(sh)} = V_i^2 B_{i(sh)} \quad (5)$$

where  $P_i$  and  $Q_i$  are the active and reactive power injected into the buses,  $P_{ik}$  and  $Q_{ik}$  are the active and reactive power flows in the lines ik,  $V_i$  and  $V_k$  are voltage magnitudes in the buses i and k,  $\theta_{ik}$  is the phase angle between voltages in buses i and k and,  $G_{ik}$  and  $B_{ik}$  are the real and imaginary parts, respectively, of admittance matrix. For the solution of the set of equations (1) to (5) for a given set of variables, the Newton-Raphson interactive method [16], as well as the Gauss-Seidel and CC Flow methods can be applied.

## 2.3. Regulation characteristics in Brazil

With a consumption of 536 TWh in 2019, Brazil ranks among the ten largest electricity consuming countries in the world [17]. To meet such demand, the Brazilian electrical system, or National Interconnected System (NIS), is characterized by a large transmission system (called the Basic Network - BN), as shown in Figure 1, with connection with almost the entire generation plants in the country. Brazilian regulation establishes, in addition to the general conditions of interface between the agents of the system, that the contracts for the purchase and sale of electricity are separated from the access and use of the transmission and distribution system. It also establishes free, non-discriminatory access to the transmission system by independent generation and distribution agents [18].

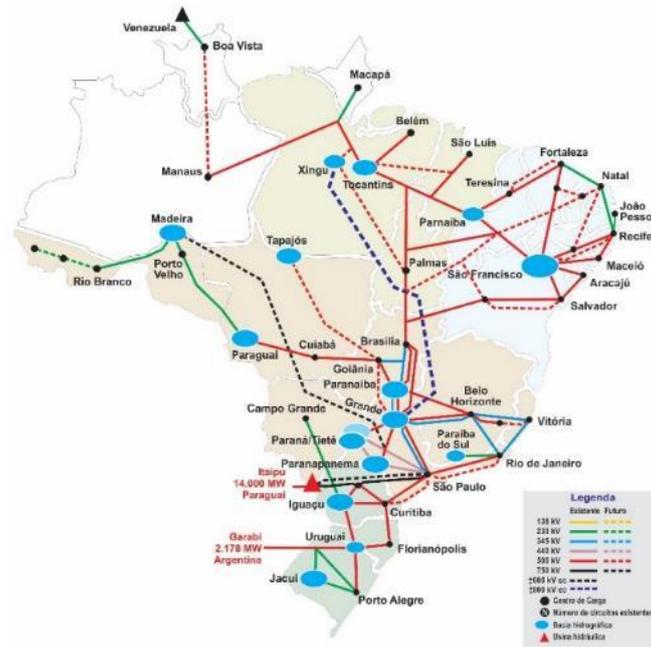


Figure 1: National Interconnected System (NIS) [19].

In Brazil, the electrical system is divided into four independent sectors: generation, transmission, distribution, and commercialization. Generation and commercialization are regulated competitive sectors, requiring installation and operation authorization, in addition to records of operations between the parties. On the other hand, transmission and distribution are regulated natural monopolies, operated by the State or through concessions and permissions [20]. Access and contracting for the use of the transmission system are regulated by the National Electric Energy Agency (ANEEL) through Normative Resolution No. 666 of 2015 [5]. Concessionaires operate in Brazil under the revenue cap regime, in which the agent is guaranteed the receipt of a regulatory revenue, called Annual Permitted Revenue (APR), regardless of the variation of the paying market. APR is calculated based on the type of installation, operating conditions, expansion planning and maintenance of the system, which is periodically reviewed [21]. The cost from transmission is recovered from the tariff for the use of the transmission system (TUST from Portuguese). TUST is divided, in the general case, in the proportion of 50% paid by the generation segment and 50% paid by consumer segment, calculated using a called Nodal method combined with a postage stamp portion [22]. Generating agents and consumers who use the transmission network are responsible for remunerating transmission concessionaires with the product

between TUST and an amount for using the system (MUST from Portuguese), called the charge for using the transmission system (EUST from Portuguese). This charge is due for each connection point between users and the Basic Network. The MUST is configured as the highest value between the maximum load contracted by the distributors and the maximum value of load measured at the connection points, annually, despite the measurements and tariffs having a monthly basis [5]. The amount of use of the system must be reported for the four years following the current year, by the electricity distributors and generators to the National System Operator (NSO), a Brazilian entity that supervises and operates the electrical system. This information is also transmitted to the Energy Research Company (ERC), a government agency responsible for planning and research in the energy sector. This information is essential for medium and long-term planning of the transmission system, due to expansion and reinforcement projects of existing lines and implementation of new facilities. In addition, APR reviews also absorb information taken from MUST to scale the appropriate remuneration by TUST. To encourage the accuracy of MUST informed by the distributors and electric energy generators, Resolution 666/2015 established penalties, defining a penalty for inefficiency. These are fines levied on the EUST in the event of over-contracting or sub-contracting. Over-contracting is understood to be the circumstance in which a distributor, for example, informs amounts greater than those measured at the connection points. Sub-contracting, on the other hand, occurs when the reported values are lower than those measured. Over-contracting has the effect of signaling to transmission system planners that there is a greater need for system expansion than the real one. As result, tariff cost is increased for all users to compensate for costs of new installations. From distributor or generator perspective, it means an increase in the charge paid, since that amount informed is the minimum amount considered in the charge account, even in cases where the measured value is lower. In turn, subcontracting can lead to system overloads, damage to equipment and an erroneous indication for planning the expansion of the system, in addition to the burden on the contractor. In both cases there is a tolerance limit of +/- 10% for the informed MUST, from which penalties are calculated [5,9,23]. The monthly charge for a given year and connection point is given by equation (6), where the value of the main charge and possible penalties for inefficiency are already counted [5].

$$E_i^{Dev} = \left\{ \sum_{m=1}^{12} [Max(C_i; X_{i,m}) \times T_i + Max(X_{i,m} - 1,1C_i; 0) \times 3T_i] \right\} + Max(0,9C_i - X_i^{Max}; 0) \times 12T_i \quad (6)$$

where  $E_i^{Dev}$  is the charge for using the transmission system due by the distributor at connection point  $i$ ;  $C_i$  is the MUST contracted by the distributor at connection point  $i$ ;  $X_{i,m}$  is the value verified by measuring electrical power at connection point  $i$ , in month  $m$ ;  $T_i$  is the tariff for using the transmission system at connection point  $i$  and  $X_i^{Max}$  is the maximum value verified by measuring electrical power at connection point  $i$  throughout the year. MUST Determination by electric power distributors and producers under Brazilian regulation presents itself as a problem of several particularities: it is essential for the planning and operation of the transmission system; enforces pressure on the tariff cost for consumers; and it also brings costs to distributors and producers, since inefficiency fines cannot be passed on to the tariffs charged to consumers [5,24]. It is a non-deterministic problem due to uncertainties associated with the process, namely: uncertainties in projection of both load and generation, as well as the network structure necessary to meet the load's growth connected to the distribution system; potential operational restrictions that imply the redistribution of power flows in the distribution and transmission networks; impact of postponement and construction delays; generation dispatch coordinated by the

agent or controlled by NSO; large consumer contracts connected to concessionaire's electricity network; penetration of new energy sources; and compatibility with the system's expansion plans. Therefore, it is possible to characterize the Brazilian MUST problem as a specific case of the broader PLF problem, in which the determination of power flows at specific points in the network under the condition of uncertainty can be modeled as a set of random variables dependent on a set of probability distributions for loads, generation and equipment. In addition, the problem moves towards a search to minimize charges to be paid and minimize the payment of penalties for inefficiency, adding an optimization step.

### 3. Proposed Methodology

The problem of determining the amounts of use of the transmission system is characterized by the need for distributors and producers to predict, with a good level of precision, the maximum active power flow at the points of connection with transmission network. Ideally, the best value to be contracted is exactly the value measured in each calculation. In this case, there would be no sub-contracting costs (such as penalties for violating upper limits contracted), nor over-contracting (with penalties for violating lower limits and payment for unused capacity) and the planning entities would have the correct signaling of future demands, being able to adapt the expansion within narrower limits. To address this specific case of the more general problem of PLF, the steps illustrated in Figure 2 were proposed. First, the problem outline condition is established, with presentation of the type of case to be addressed, simplifications and assumptions considered. Next, generation, load and grid uncertainties are modeled within PLF in form of probability distributions as discussed in Section 2.1. After modeling uncertainties, the following parameters are established: loads, generations, and tariffs. From these definitions, PLF simulation is then performed, where the data previously organized is used as input to the load flow simulation software and classified according to the amount of the charge due. Finally, results obtained are analyzed and discussed, noting their limitations.

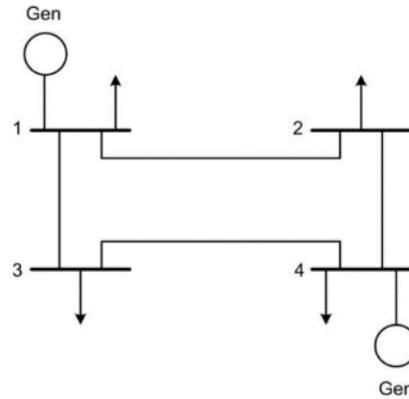


**Figure 2:** Steps of the proposed methodology.

#### 3.1. Boundary conditions

The problem of static load flow analysis is well known and well established in the literature. As already discussed, the probabilistic nature of the load and generation in solving that problem is also extensively addressed. Therefore, it is feasible to approach the problem from the PLF, which essentially transforms the random variables of a system's inputs, defined in terms of probability density functions, into random variables in the output, also defined in terms of PDF [13]. Figure 3 illustrates the single-line diagram from a very classic case of a small system with 4 buses and 2 generators presented in [16]. In DLF, each element of this system has discrete values for the parameters of load, generation, and network, such as active and reactive power of loads and generators and line admittance matrix. On the other hand, for the execution of PLF, each parameter relevant

to the approach of the proposed problem is considered as a set of random variables. Inserting the specific situation of Brazilian regulation, it is not necessary to take reactive powers into account. Also, network uncertainties caused by temperature changes are not considered, so the admittance matrix is composed of a set of constant values.



**Figure 3:** 4 buses and 2 generators system presented by [16].

Thus, in the present paper, the following sets of random variables, equations (7), (8) and (9) are considered, based on the example illustrated in Figure 3:

$$\Omega_D = \{Pd_1, Pd_2, Pd_3, Pd_4\} \tag{7}$$

$$\Omega_G = \{Pg_1, Pg_2, Pg_3, Pg_4\} \tag{8}$$

$$\Omega_{branch} = \{S_{1-2}, S_{1-3}, S_{2-4}, S_{3-4}\} \tag{9}$$

where  $\Omega_D$  is the set of random variables of the active powers demanded in the buses,  $Pd_i$  is the random variable of the active power demanded in the bus  $i$ ,  $\Omega_G$  is the set of random variables of the active powers generated in the buses,  $Pg_i$  is the random variable of the active power generated in the bus  $i$ ,  $\Omega_{branch}$  is the set of random variables available on the network lines,  $S_{i-k}$  is the binary random variable that represents the availability, or status, of the line connecting the bus  $i$  to bus  $k$ . The powers are given in MW and the status is a dimensionless variable. Charges are calculated by equation (6) for a given connection point in Brazil's specific case. Each connection point has a set of parameters necessary for equation (6). Table 1 shows fictitious data for the system in Figure 3 if buses 2 and 3 are connection points between the electrical network of a distributor and the electrical transmission network.

For the case exemplified in Table 1, there would be a penalty for subcontracting in bus 1 for exceeding the measured value  $X_2$ , above the limit of 10% of the contracted amount  $C_2$ , exceeding the tolerance. In bus 3, the charge  $C_3$  would be due, since the measured value  $X_3$  is less than the contracted, but still within the tolerance limit for over contracting.

**Table 1:** Connection point characteristics with contracted amount, measured amount, and usage tariff as input data.

Bus	Contracted amount ( $C_i$ ) MW	Measured amount ( $X_i$ ) MW	Usage tariff ( $T_i$ ) \$/MW	Charge due ( $E_i^{Dev}$ ) \$
2	170	190.4	50	10,030.00
3	200	194.0	45	9,000.00

### 3.2. Modeling uncertainty variables

As previously discussed, for PLF, loads, generators and network conditions are not treated as deterministic inputs. In this case, these parameters are described as random variables described by probabilistic distributions capable of reflecting the type of uncertainty of the object studied, be it loads, generators or network components. For loads connected to the buses, their active powers will be expressed by normal Gaussian distributions, with known means and standard deviations [13]. Generators of traditional sources of energy will also be random variables of normal profile. As previously discussed, the wind speed and the load distribution generated at points with a high concentration of wind generators can be modeled as a two-parameter Weibull distribution [7,14,15]. Thus, generation points connected to the buses with a high concentration of wind generators will be considered as Weibull distribution with two parameters, the first defining the scale and the second defining the shape of the curve. The transmission network, distribution networks and their interconnections are composed of various equipment such as cables, towers, transformers, circuit breakers, among others. Uncertainties associated with network parameters, i.e., temperature, will not be considered in this paper and adopted as constant. Uncertainties related to the topology will be expressed through availability of some equipment that make up the network. Availability is the property of an item that, when used under conditions established in an ideal support environment, will be operational at a given time [25]. Thus, the availability of an item during a given time interval, can be expressed from equation (10):

$$A_j = \frac{u_j}{u_j + d_j} \tag{10}$$

where  $A_j$  is the equipment's availability  $j$ ,  $u_j$  is the time in which the equipment is available and in operation and  $d_j$  is the time in which the equipment is unavailable and out of operation. It is not the scope of this paper to discuss the determination of the reliability and availability of equipment, the concept that availability represents a fraction of time that an item is operating in function of the calendar. In this way, topology uncertainties will be represented by rectangular or constant distributions, in which the probability of an equipment being in operation is equal to its availability. Which will be treated as a parameter known to the system operator. The equipment considered will be lines that connect buses, with other items considered as perfect availability equal to 1. In this way, it is possible to simulate contingencies in the lines, such as works, maintenance, vandalism, and other aspects. Table 2 shows an example of model data for the system buses in Figure 3 for the PLF, with the column type of PDF for  $PD_i$  a definition of which distribution was chosen to model the load connected to the bus,  $\mu PD_i$  or  $\eta PD_i$  are the mean or the scale for normal distribution or Weibull, respectively and  $\sigma PD_i$  or  $\beta PD_i$  are the

standard deviation or shape, also for normal distribution or Weibull respectively. In the sequence there is the same data structure representing the generators connected to the buses.

**Table 2:** Model data for loads and generators connected to system buses.

Bus	PDF type for $\mu PD_i$ or $\eta PD_i$	$\sigma PD_i$ or $\beta PD_i$	PDF type for $\mu PG_i$ or $\eta PG_i$	$\sigma PG_i$ or $\beta PG_i$		
1	Normal	50.0	2.5	Normal	180.0	9.0
2	Normal	170.0	8.5	-	0	0
3	Normal	200.0	10.0	-	0	0
4	Normal	80.0	4.0	Weibull	318.0	2

Data presented in Table 2 are in MW, except for the parameter  $\beta$ , which is dimensionless. Table 3 expresses model data for uncertainties of the system lines in a similar way to that performed for loads and generators. The availability column shows the probability that the equipment will be in operation at any time.

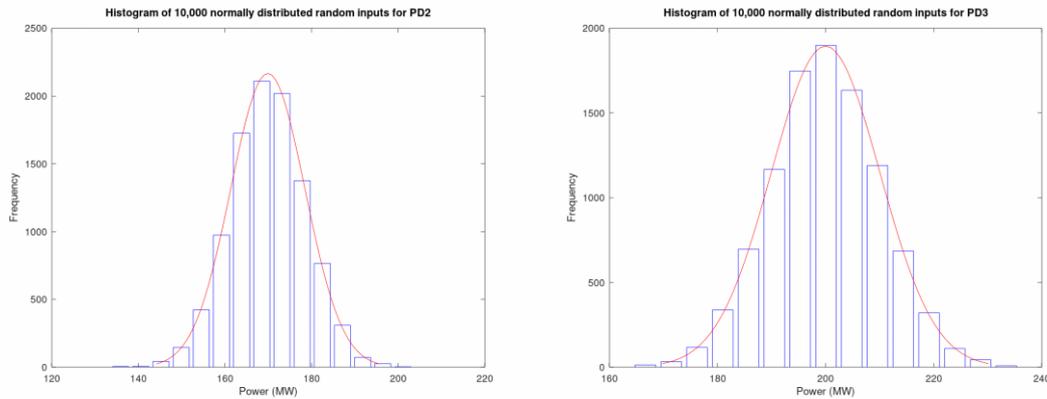
**Table 3:** Model data for connection lines between buses.

Line	PDF type for $S_{i-k}$	Availability
1-2	Rectangular	0,99670
1-3	Rectangular	0,99850
2-4	Rectangular	0,99590
3-4	Rectangular	0,99790

### 3.3. PLF simulation

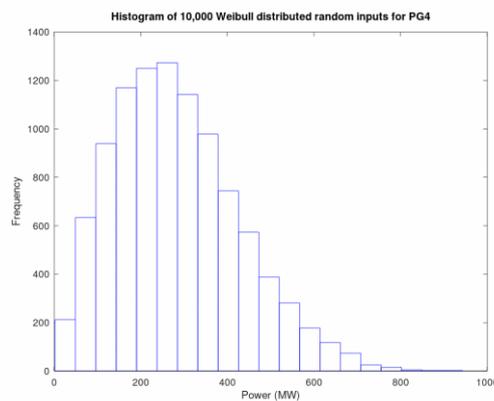
As presented in Section 2.2, there are two approaches to the PLF problem: analytical and numerical. In the present paper, it was decided to use the numerical method, i.e., Monte Carlo. This choice is due to the Monte Carlo method being used as a reference in the comparison of other PLF methods due to its precision [13]. Despite the greater computational intensity, in the specific case of Brazilian regulation for connection to the Basic Network, many calculations are not necessary in a short period, since amounts must be reported annually. Thus, a longer computational effort time is tolerable. Briefly, PLF by numerical method is the repetition of deterministic method for many times. In this sense, as a deterministic method, the Newton-Raphson method was used through the application Matpower [26] running on GNU Octave software. In this configuration, there is the solution of the set of non-linear equations (1) to (5) by the iterative method until a determined error, in this case  $10^{-8}$ . The solution will be reached for each scenario generated by a pseudorandom number generator, which randomly selects one of the data entry possibilities from the PDF defined in the previous step. For the example illustrated in Figure 3, bus 2 has a load that, according to Table 2, has a normal distribution, with an average  $\mu PD_2$  of 170 MW and standard deviation  $\sigma PD_2$  of 8.5 MW. Applying the inverse normal function over a random number between 0 and 1, one possible sample is obtained within that distribution. Figure 4 illustrates

the histogram in 15 classes created from 10,000 random numbers normally distributed with the parameters of loads connected to bus 2 and bus 3, with the extreme results being concentrated in the histogram's outer classes. Note the higher frequency in average vicinity determined in Table 2.



**Figure 4:** Histograms of the active powers of the loads on buses 2 and 3 simulated by normally distributed pseudo-random numbers.

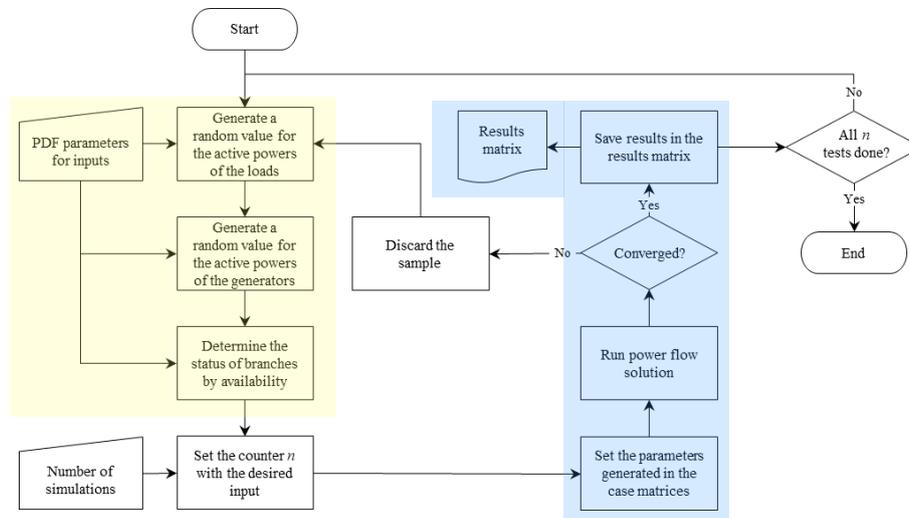
PDF for 10,000 simulations for the generation connected to bus 4 with a high presence of wind generators is shown in Figure 5. Due to the type of distribution used, Weibull with 2 parameters, it is possible to observe some of the typical characteristics of this kind of histogram: non-symmetry and an elongated “tail” in one direction.



**Figure 5:** Histogram of active power in the generator of bus 4 simulated by pseudo-random numbers distributed by a 2-parameter Weibull distribution.

A simplified flowchart of the algorithm for applying the Monte Carlo method to the PLF problem is shown in Figure 6. The section highlighted in yellow represents mainly those steps of obtaining the random variables of load inputs, generation, and network parameters. Then a counter with the number of simulations is established, higher the number being, greater the precision of the simulation, but greater the computational effort. For this paper  $n$  was determined in 10,000. Additionally, the zone highlighted in blue composes the main stages of DLF by the Newton-Raphson method considered. These steps include updating the Matpower input matrices for the

desired case, the DLF solution, evaluating the convergence (and discarding the sample if it fails this test) and recording the results in the results matrix. Finally, after the  $n$  simulations, the results evaluation stage is then performed.



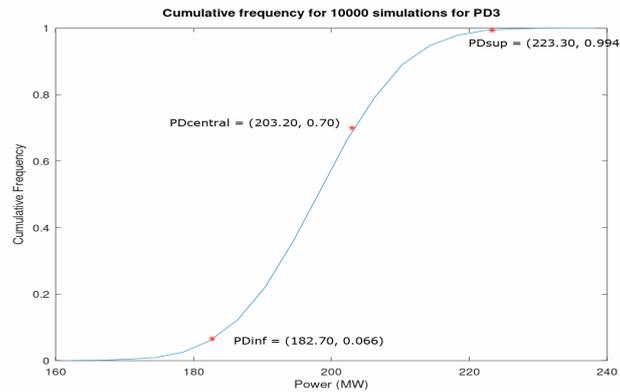
**Figure 6:** Descriptive flowchart of numerical simulation for PLF.

### 3.4. Results analysis

Considering the bus results of PLF, it is possible to advance on the specific case of Brazilian regulation. There are two main ways to approach the amount contracted at the points of connection with the RB: by determined risk or by minimizing the charge [11]. In the first case, the main concern is to know the probability of incurring penalties for subcontracting and subcontracting. However, there is no concern about the amount of the charge paid which may be greater than necessary. In the second case, there is a quest to minimize charges paid based on the PLF. The lower charge is associated with a probability that may or may not be acceptable to generators and distributors. An approach that considers the search for minimum burden within a predetermined risk range is also possible. To exemplify the amount per determined risk, consider the PDF for  $PD_3$ , shown in Figure 4, this random variable representing the result of the PLF of this bus and that bus is a connection point between the distribution network and the transmission network. Thus, the cumulative distribution function (CDF) can be plotted for the data set as shown in Figure 7. In this situation, the 203.2 MW heap was contracted from the transmitting agent. This value represents that 70% of measurements will probably be below that number and 30% will be above. Considering that there is a tolerance of +/- 10% before there are penalties for over-contracting and sub-contracting, one can find the pairs of coordinates that present the possibility of being outside these limits. For the upper limit of 223.3 MW there is a chance of less than 0.6% of higher values occurring. As with the lower limit, there is a risk that the measurement will be below 182.7 MW of approximately 6.6%. However, it is important to note that although the risks of exceeding the limits are small, the selected value does not necessarily provide the lowest cost, since there are great chances of both over-contracting and sub-contracting within the limits without penalty.

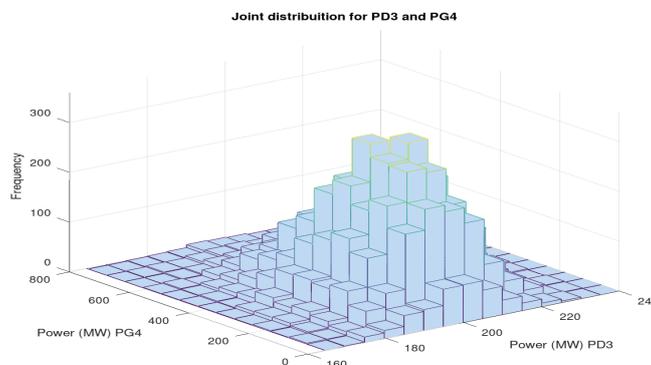
This approach to the problem of the contracting amount is not the focus of this paper, however it will be used to

measure the risk assumed when obtaining the amount by the second method: the one with the lowest value. The selection of the amount for the lowest charge seeks to find the amount to be contracted with the greatest probability of occurring, which, consequently, would result in the lowest charge due by the distributor or generator agent. The method described by [11,27] can be synthesized in two stages: first, the probability distributions obtained in the simulations by the PLF in the connection buses are related by means of a joint probability distribution which can be described as a function probability mass.



**Figure 7:** Cumulative distribution function of active power  $PD_3$  for determination by assumed risk.

Then, each combination between the central values of the probability histogram of each bus has its cost calculated and weighted by the joint probability previously obtained. Finally, the smallest of the charge values found provides the set of amounts to be contracted in each bus. Taking the load  $PD_3$  connected to bus 3 and generator  $PG_4$  connected to bus 4, as random variables and their respective probability distributions shown in Figure 4, it is possible to know the joint probability of both distributions. Figure 8 illustrates a graphical representation as a function of absolute frequency for 10,000 simulations and 15 bins. The central value of each bin is then used as a possible amount to be contracted for that connection point, the same concept being applicable to all other connection points. Thus, assuming there are 2 connection points and 15 central values of the histogram containers, there will be a total of  $C_n$  different combinations of choices. For this case,  $15^2$  would result in a total of 225 possibilities.



**Figure 8:** Probability mass function of joint distribution from active power  $PD_3$  and  $PG_4$ .

For each contracting alternative, the weighted charge due is calculated based on the joint probability of the alternatives. The process is repeated for all different Cn possible combinations. Then, the search for the lowest expected charge within this set of results is carried out. This minimum value represents a set of amounts in each bus, which represent the lowest value and have an associated risk.

#### 4. Tests and results: Modified IEEE 14 bus system

To validate the proposed methodology, a case study was carried out using the IEEE 14 bus system [28] with some modifications that will be detailed below. The single-line diagram is shown in Figure 9. The area highlighted in yellow represents, in this case, the region understood by the distributor. Buses 6 and 9 will be considered as connection points between the distributor and the Basic Network (transmission network). The other lines outside the highlighted region are understood as part of the transmission network. A change to the original system is the insertion of a wind generator in bus 12, in the area of the electrical distribution network. In addition, the existence of wind generation was considered next to bus 2.

The input random variables for this system are presented in equations (11) to (13).

$$\Omega_D = \{Pd_1, Pd_2, \dots, Pd_{14}\} \tag{11}$$

$$\Omega_G = \{Pg_1, Pg_2, \dots, Pg_{14}\} \tag{12}$$

$$\Omega_{branch} = \left\{ S_{1-2}, S_{1-5}, S_{2-3}, S_{2-4}, S_{2-5}, S_{3-4}, S_{4-5}, S_{4-7}, S_{4-9}, S_{5-6}, S_{6-11}, S_{6-12}, S_{6-13}, S_{7-8}, S_{7-9}, \dots \right. \\ \left. \dots S_{9-10}, S_{9-14}, S_{10-11}, S_{12-13}, S_{13-14} \right\} \tag{13}$$

Models for load and generation uncertainties are shown in Table 4, with active powers generated in buses 2 and 12,  $PG_2$  and  $PG_{12}$  respectively, modeled as Weibull distributions with two parameters due to high concentration of wind energy on those generators. Standard deviations vary between 5 and 10% randomly. The uncertainties of the electrical network regarding availability are shown in Table 5. All lines were modeled as rectangular distributions and were randomly generated within the limit between 0.9950 and 1. Availability affects the line's binary parameter status. When line status is equal to 1 it is available and when the status is equal to zero, i.e., when the line is unavailable, it is removed from the DLF.

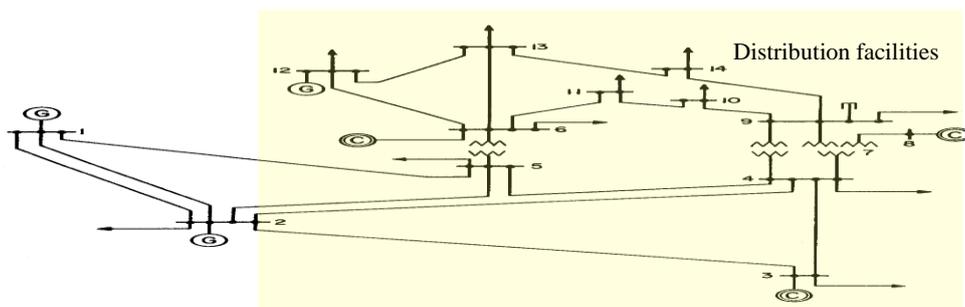


Figure 9: Single-line diagram of the modified IEEE 14 bus system.

**Table 4:** Data for modeling loads and generators connected to buses of modified IEEE 14 bus system.

Bus	PDF type for $PD_i$	$\mu PD_i$ or $\eta PD_i$	$\sigma PD_i$ or $\beta PD_i$	PDF type for $PG_i$	$\mu PG_i$ or $\eta PG_i$	$\sigma PG_i$ or $\beta PG_i$
1	Normal	0	0	Normal	232.40	11.6
2	Normal	21.70	1.10	Weibull	40	2
3	Normal	94.20	4.70	-	0	0
4	Normal	47.80	4.60	-	0	0
5	Normal	7.60	0.38	-	0	0
6	Normal	11.20	0.56	-	0	0
7	Normal	0	0	-	0	0
8	Normal	0	0	-	0	0
9	Normal	29.50	2.20	-	0	0
10	Normal	9.00	0.45	-	0	0
11	Normal	3.50	0.18	-	0	0
12	Normal	6.10	0.30	Weibull	30	1.75
13	Normal	13.50	1.00	-	0	0
14	Normal	14.90	0.75	-	0	0

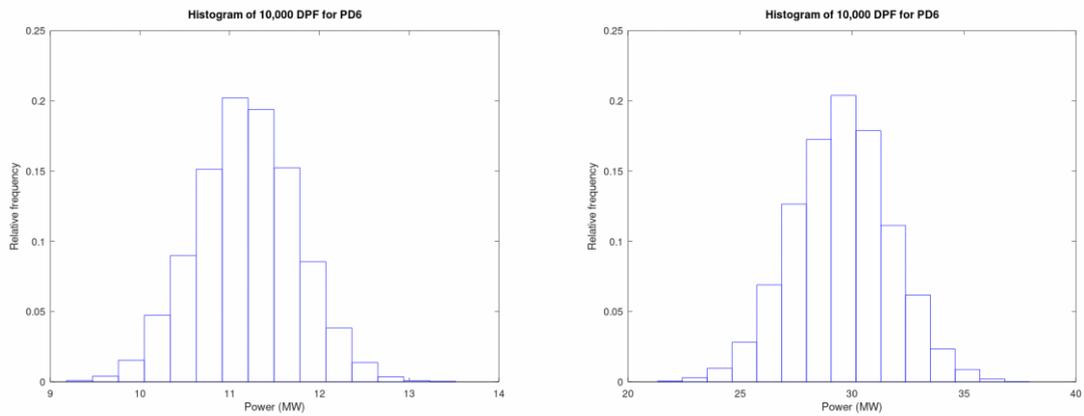
In Table 4, all data are in MW, except for parameter  $\beta$ , which is dimensionless.

**Table 5:** Data for modeling connection lines between buses for modified IEEE 14 bus system.

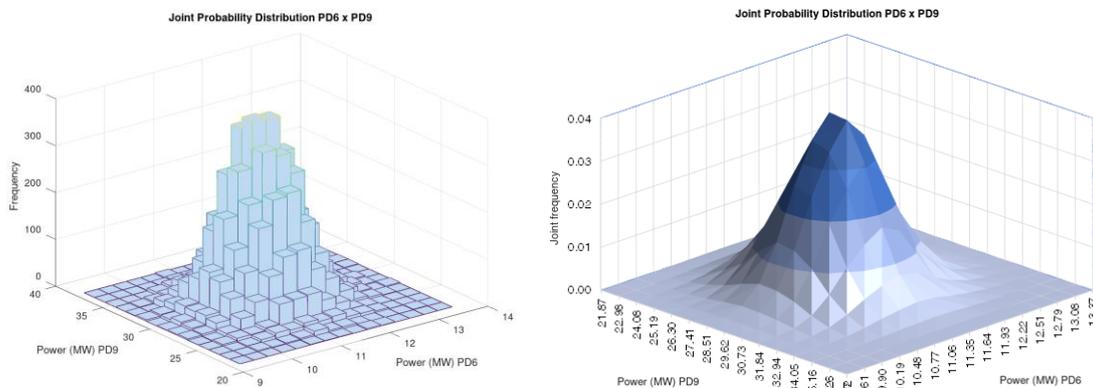
Line	PDF type for $S_{i-k}$	Availability	Line	PDF type for $S_{i-k}$	Availability
1-2	Rectangular	0,99670	6-11	Rectangular	0,99670
1-5	Rectangular	0,99850	6-12	Rectangular	0,99950
2-3	Rectangular	0,99590	6-13	Rectangular	0,99820
2-4	Rectangular	0,99790	7-8	Rectangular	0,99880
2-5	Rectangular	0,99750	7-9	Rectangular	0,99810
3-4	Rectangular	0,99790	9-10	Rectangular	0,99750
4-5	Rectangular	0,99500	9-14	Rectangular	0,99660
4-7	Rectangular	0,99650	10-11	Rectangular	0,99790
4-9	Rectangular	0,99590	12-13	Rectangular	0,99760
5-6	Rectangular	0,99930	13-14	Rectangular	0,99920

Figure 9 shows in both histograms results from connection buses 6 and 9, left and right, respectively. These results are composed from a total of 10,000 simulated DLF, in a time of 195 seconds, divided into 15 bins of equal width. Extreme results were grouped at the outer limits. The results for  $PD'_6$  have an average equal to a  $\mu PD'_6 = 11.19$  MW with standard deviation  $\sigma PD'_6 = 0.561$  MW and the results for  $PD'_9$  have an average equal to  $\mu PD'_9 = 29.54$  MW with standard deviation  $\sigma PD'_9 = 2.200$  MW. Distributions were combined to find the joint

mass distribution, which is composed of the product of the relative frequency of each bin that make up the histogram with the results  $PD'_6$  and  $PD'_9$ . Figure 10 illustrates the joint probability of the PLF in buses 6 and 9, with the graph on the left showing distribution in absolute frequency and the right in relative frequency. The highest points in these graphs represent the combinations of active power most likely to occur on buses 6 and 9. To simplify the search for the lowest charge, ranges of values with lowest relative frequencies were discarded, concentrating the search around most frequent values. Thus, instead of  $15^2$  alternatives,  $8^2$  alternatives were considered, which corresponds to 96.1% of cases for bus 6 and 95.2% of cases for bus 9. Tariffs  $T_6$  and  $T_9$  that were needed to calculate  $E_1^{Dev}$  given by equation (6) were \$ 45 and \$ 50, respectively.



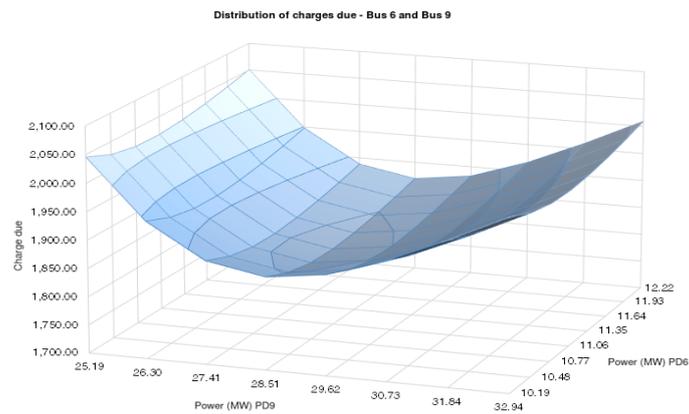
**Figure 9:** PLF results after 10,000 simulations for buses 6 and 9 given in relative frequency.



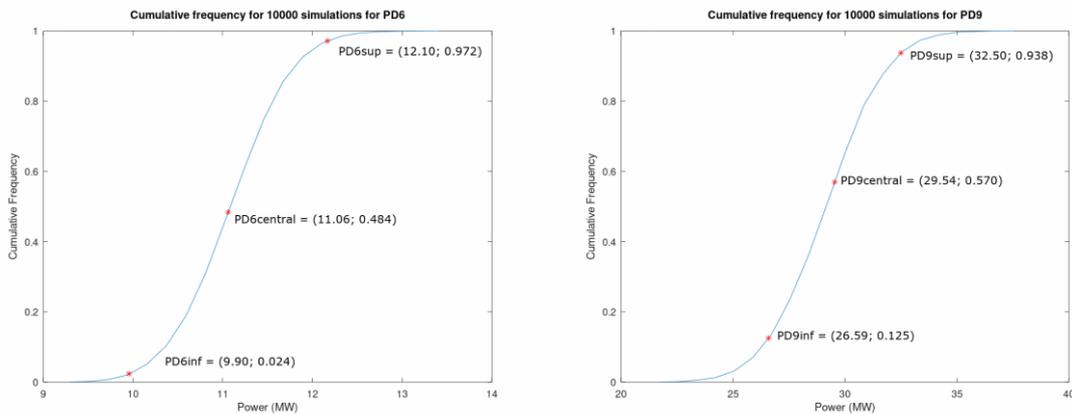
**Figure 10:** Probability mass function of the joint distribution of the active power  $PD_6$  and  $PD_9$ . On the left the data are presented in absolute frequency and on the right in relative frequency

The set of charges as a function of the joint probability of buses 6 and 9 is illustrated in Figure 11. The objective is to minimize the charge due and it is necessary to find the minimum point of the function shown. As it is a discrete distribution, a simple search for the lowest value within the matrix that accumulates the data brought the value of \$1,836.17, referring to the suggested amounts  $C_6 = 11.06$  MW and  $C_9 = 28.51$  MW. This set of values represents the most likely and, therefore, the lowest cost amounts to be contracted. Both values obtained in Figure 11, contains a risk associated with its choice, as detailed at the beginning of this section, and are shown

in Figure 12 for  $C_6$  on the left and  $C_9$  on the right. For  $C_6 = 11.06$  MW, there is a probability of approximately 48.4% of the measured amount being less than the contracted and 2.4% chance of penalty for over-contracting. On the other hand, there is approximately 52.6% probability that the amount measured will be higher than the contracted amount and 2.8% chance of having a penalty for sub-contracting. For  $C_9 = 29.54$  MW, there is a probability of approximately 57.0% of the measured amount being less than the contracted and a 12.5% chance of penalty for over-contracting. On the other hand, there is approximately 43.0% probability that the amount measured will be higher than the contracted amount and 6.2% chance that there will be a penalty for subcontracting. When there is a greater dispersion of data, represented by a proportionally greater standard deviation, the risk in hiring is also increased, as it is possible to notice in the comparison between buses 6 and 9.



**Figure 11:** Distribution of charges according to the amounts contracted in buses 6 and 9.



**Figure 12:** Cumulative density function of active power  $PD_3$  for determination by assumed risk.

## 5. Conclusion

Unbundled electrical systems present challenges for operators and agents. The regulation on transmission, a typical natural monopoly, needs to balance the adequate remuneration to the network operator, allow free non-discriminatory access and seek reasonable tariffs. In this context, it is important that the quality of information on the use of the system is the best possible. This paper sought to present a way of inserting the inherent

uncertainties of loads, generators and network equipment in the probabilistic load flow problem, in order to insert generators with a high presence of wind energy and the availability of equipment, through numerical simulations, e.g., Monte Carlo. From these simulations, a specific case of Brazilian regulation was presented to illustrate a real situation of interface between distributors and generators with the transmission network. The results obtained in the modified IEEE 14 bus system, in the form of active power flows in buses 6 and 9, allow the distributor to contract the appropriate amount to minimize the charge due, provided he accepts the risk incurred. This way of approaching the Brazilian problem presented in this work can be expanded to other situations of medium and long-term planning of the interface between the transmission and its users.

## **6. Recommendations**

The method of determining the use of the Electricity Transmission System by means of FPP and Monte Carlo numerical simulation is recommended for medium and long term planning of the operation, maintenance and expansion of the network, by operators, owners, users, government and inspectors. Application in short-term or fast-changing situations may not be adequate, given the computational intensity required, may not be suitable for these scenarios.

In addition, this article recommends further work to:

- Quantify the influence of photovoltaic generation and distributed generation in points of interest;
- Compare computational efforts of other DLF methods;
- Insert uncertainties of ancillary equipment and other devices in addition to the lines themselves, such as, for example, transformers.
- Compare the methods proposed here with real data.

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