American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)
ISSN (Print) 2313-4410, ISSN (Online) 2313-4402

http://asrjetsjournal.org/

Optical Phenomena in Time Dependent Medium

Cícero Julião*

Instituto Federal de Alagoas, BR-104, 111, Murici-AL, 57820-000, Brazil
Email: cicero.junior@ifal.edu.br

Abstract

How to deal with optical phenomena if the physical quantities are time-dependent? When a light wave propagating in space meets an interface between two media, a transmitted and a reflected wave appears. However, if a medium abruptly changes the value of its dielectric constant, even without an interface dividing space, we also have the phenomenon of reflection and refraction. Thinking of time as a coordinate similar to the spatial coordinates, the interface found also provides a change in the medium. But a change in time. Thus, known relationships, such as Snell's Law, should be reviewed for such phenomena. This article deals with some situations where we have non-fixed dielectric constants, changing with time. From Maxwell's equations, we demonstrate how to simulate the propagation of an electromagnetic wave in a medium that varies its dielectric constant over time. We used the finite difference method in the time domain (FDTD). We show the interesting phenomenon of temporal refraction and reflection.

Keywords: Photonic Time Crystal; Periodic; Maxwell Equations; Dielectric Constant.

1. Introduction

Some optical phenomena are elementary. One of these is known as Snell's Law. The simplicity of its observation and the consequent simplicity in understanding its essential points is so great that we can deal with it in elementary school classes. Even in these classes, we can trust that their understanding is possible. Snell's Law deals with the simple situation of a light ray reaching the boundary between two optical media. Snell's Law describes the change in the direction of propagation. In this process, the energy is conserved, but the component of the moment perpendicular to the boundary is not [1].

----

* Corresponding author.

We have an important statement here. The change in the optical medium occurs in space. There is a change in the spatial structure of the material. However, what happens if the change does not occur in space, but intime? That is, what if the optical medium undergoes temporal change? It is the case for so-called *active medium* [2]. An active (or dynamic) medium is one where the dielectric constant depends on time. That is, an active material has a function $\epsilon(t)$, being non-homogeneous in time [3–6]. It is effortless to find literature dealing with functions of type $\epsilon(r)$, where the homogeneity breaking occurs spatially; but its temporal counterpart is not so conventional. However, functions of type $\epsilon(t)$ have been studied since the 1950s [7]. More recent attempts to produce active materials include the use of thermal variation, mechanical control of materials, and resonant cavities [8–10]. In this paper, the approach is about an effect of temporal change in an optical medium: the appearance of reflected and refracted waves arising from the temporal variation of the dielectric constant. The consequence of such waves is the possibility of permissible and prohibited bands for wave vectors in the propagation of electromagnetic waves [11]. Temporal reflection and refraction are already well established in the literature. In this paper, we show the pillars for the realization of these phenomena and how to develop computational simulations [1]. This first section shows an introduction to the main ideas. Section 2 deals with the phenomena of temporal reflection and refraction, responsible for several other effects for active materials. Section 3 analyzes the Finite Time Domain Differences Method (FDTD) used in computational simulations. Section 4 give us an example of wave propagation inside an active medium. In section 5 we present our conclusions and discuss possible practical applications.

2. Reflection and Refraction

Let us imagine a ray of light propagating in any uniform medium. Suddenly, this optical medium changes. Its dielectric constant changes drastically. As a result, the propagation of light changes. However, this time, because of the temporal change. In the literature, such a change is called temporal refraction. If we think of time as a further propagation coordinate, we have another analogy with the spatial case. There is a contour in the temporal dimension of propagation. So, the light finds such a contour, producing a (temporal)refraction [1]. As we will see now, such a change is accompanied by another optical phenomenon: a reflection. Still, while the refraction here is seen in the temporal domain (with the change in frequency), the reflection is seen from the spatial point of view. A reflected wave in space appears as soon as the light reaches the boundary in the temporal dimension [12]. In order to understand such unfolding, our starting point will be the equation governing the propagation of a wave in a homogeneous medium from Maxwell's formalism, i.e.

$$\nabla^2\vec{E} - \frac{n^2}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0. \qquad (1)$$

Eq.(1) can be satisfied by different families of solutions [13]. We will use plane waves,

$$\vec{E}(\vec{r},t) = \overrightarrow{E_o}e^{i(\vec{k}t-\omega t)} \qquad (2)$$

Remember that k relates the wave vector and frequency,

$$k = \frac{\omega}{c}n \qquad (3)$$

In the spatial case of an interface between the two optical media at $x = x_0$, of refractive indices $n_1$1 and n_2, there is a transmitted wave and a reflected wave. In medium 1, for n_1, the total electric field will be the sum between the incident wave and the reflected wave. In the medium 2, for $x > x_0$, we have only the refracted wave.

2.1. Temporal reflection

For the temporal case, however, the first significant difference arises. Let us take the boundary between the two mediums at $t = t_0$. We are now dealing with the temporal case. So, at a certain instant t_0, the optical medium changes. In this way, for $t < t_0$ we will have a single wave, the incident one. However, in the medium 2 ($t > t_0$), we have two waves: one transmitted and the other reflected - in space. The emergence of a reflected wave in space under a temporal change, though perhaps surprising, is seen when we analyze the conditions of continuity in the vicinity of $t = t_0$. The continuity conditions for the case of spatial changes are well treated in the literature. So let us think about the case of temporal changes, the focus of our approach.

Maxwell's equations have temporal derivatives just in two of the four equations.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \qquad (4a)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}. \qquad (4b)$$

To ensure continuity, we need to verify such equations in the neighborhood of t_0. Before $t = t_0$, we will have the relations below.

$$B = \mu_1 H_1 \qquad (5a)$$

$$D = \epsilon_1 E_1 \qquad (5b)$$

After $t = t_0$, the relations remain, changing only under the new characteristics of the material:

$$B = \mu_2 H_2 \qquad (6a)$$

$$D = \epsilon_2 E_2 \qquad (6b)$$

The more general solutions for the fields must admit propagating wave components in both direction:

$$\overrightarrow{H_2} = \overrightarrow{H_2^{\rightarrow}} + \overrightarrow{H_2^{\leftarrow}} \qquad (7a)$$

$$\vec{E} = \overrightarrow{E_2^{\rightarrow}} + \overrightarrow{E_2^{\leftarrow}} \qquad (7b)$$

In the contour of $t = t_0$, equations 6 and 7 lead to

$$\vec{B} = \mu_1 \overrightarrow{H_1} = \mu_2\left(\overrightarrow{H_2^{\rightarrow}} - \overrightarrow{H_2^{\leftarrow}}\right) \quad (8a)$$

$$\vec{D} = \epsilon_1 \overrightarrow{E_1} = \epsilon_2\left(\overrightarrow{E_2^{\rightarrow}} + \overrightarrow{E_2^{\leftarrow}}\right) \quad (8b)$$

Solving (8) above for the $\vec{E}$ fields, the solution gives us [6]:

$$\overrightarrow{E_2}^{\rightarrow} = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} + \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}\right)\overrightarrow{E_1} \quad (9a)$$

$$\overrightarrow{E_2}^{\leftarrow} = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} - \sqrt{\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}}\right)\overrightarrow{E_1} \quad (9a)$$

Of course, if $\epsilon_1 = \epsilon_2$ and $\mu_1 = \mu_2$, there is no reflected wave ($\overrightarrow{E_2^{\leftarrow}} = 0$). this case is not attractive. We are interested in active materials. Therefore, the equations (9) indicate the existence of a reflected wave after time $t = t_0$. For nonmagnetic materials, from equations (9) we have

$$\overrightarrow{E_2}^{\leftarrow} = \frac{1}{2}\left(\frac{\epsilon_1}{\epsilon_2} - \sqrt{\frac{\epsilon_1}{\epsilon_2}}\right)\overrightarrow{E_1}. \quad (10)$$

Equation 10 shows the relation between the two propagating waves.

2.2. Temporal refraction

Well known in the literature, Snell's Law gives us the relation between the refractive indices of the media and the angles involved in incidence and refraction [1]:

$$n_1 sen\Theta_1 = n_2 sen\Theta_2. \quad (11)$$

We need to observe Eq. (3) for the frequency since the phenomenon analyzed is in the temporal domain. Therefore, such a dispersion relation must satisfy [14]:

$$\omega_i = \frac{kc}{n_1}. \quad (12a)$$

$$|\ \omega_r\ | = \omega_t. \quad (12b)$$

$$\omega_t = \omega_i \frac{n_1}{n_2}. \quad (12c)$$

We can see from Eqs. (12) that the reflected and transmitted waves have the same modulus in frequency. Therefore, for such waves to be distinct, it is necessary $\omega_t = -\omega_r$. From Eq. (2) we see another reason to emerge a reflected wave: the negative frequency wave has is the positive frequency wave's form but propagating in the opposite direction. We can also notice that a reflected wave in time would not make sense. A temporal reflection (i.e., a wave propagating in the negative direction of the time axis) would be a forwarding of information from the future to the past.

Also, from Eqs. (12) we arrive at the following dispersion relation:

$$\omega_i n_1 = \omega_r n_2. \tag{13}$$

Equation (13) can be compared to Snell's Law, but valid for temporal refraction. Therefore, the phenomena of reflection and refraction, already well discussed for the spatial case, have their parallel to the temporal case. In the next section, we show how to implement a computational simulation for reflection observation and temporal refraction.

3. Simulation in Time Domain

We used the finite-differences time-domain method (FDTD) to simulate the phenomena described in the previous section. First, we see how to transfer the propagation equations of the continuous electromagnetic fields to the discrete case, required for use on computers. Then, in the next section, we will introduce the change in time fo the code. The use of the FDTD method is highly helpful for the learning of physical phenomena. For instance, in electromagnetism, we can directly use Maxwell's equations and directly treat them, with almost no adaptation - except, of course, their discretization. The FDTD method directly implements the equations that govern the phenomena [6,15].The FDTD method uses finite differences to compose approximations. In Maxwell's equations, both the spatial derivatives and the temporal derivatives are in the form of differences. Consider the following Taylor Series expansions for the function f(x) at x_0:

$$f\left(x_0 + \frac{\delta}{2}\right) = f(x_0) + \left(\frac{\delta}{2}\right) f'(x_0) + \frac{1}{2!} f''(x_0) + \cdots, \tag{14a}$$

$$f\left(x_0 - \frac{\delta}{2}\right) = f(x_0) - \left(\frac{\delta}{2}\right) f'(x_0) + \frac{1}{2!} f''(x_0) + \cdots, \tag{14b}$$

Subtracting the first from the second equation and dividing by δ, we obtain

$$\frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta} = f'(x_0) + \frac{1}{3!}\frac{\delta^2}{2^2} f'''(x_0) + \cdots. \tag{15}$$

The relevance of the Eq. (15) is: the derivative added to terms with δ of order greater than or equal to two equals a simple difference. Taking δ sufficiently small, we can ignore the terms involving δ^a for $a \geq 2$, and we get the not surprisingly simple result (already known from the first steps in differential calculus):

$$f'(x = x_0) \approx \frac{f\left(x_0+\frac{\delta}{2}\right)-f\left(x_0-\frac{\delta}{2}\right)}{\delta}. \tag{16}$$

With this information in hand, we can implement the algorithm suggested by Kane Yee in 1966. In summary, these are the steps [16]:

1. To replace the derivatives by finite differences, discretizing both space and time (in this way, it is

possible to schematize the electric and magnetic fields as interspersed in space and time)

2. Rewrite the differences as updating equations (the past fields are the base for the future fields)
3. Calculate the magnetic field just one step throughout the computational space, making the magnetic field known a step in the future.
4. Calculate the electric field just one step throughout the computational space, making the electric fi-eld known a step in the future.
5. Calculate the electric and magnetic fields, according to the previous two steps, until the end of the simulation time.

For discretization of the space, in our simulations, we consider the points where we calculate the electric field to the left of the points where we calculate the magnetic field. With the first two steps, we can arrive at the equations of the fields below [17].

$$E_z^{n+1}[x] = E_z^n[x] + \frac{\Delta t}{\epsilon \Delta x}\left(H_y^{n+\frac{1}{2}}\left[x+\frac{1}{2}\right] - H_y^{n+\frac{1}{2}}\left[x-\frac{1}{2}\right]\right) \qquad (17)$$

$$H_y^{n+\frac{1}{2}}\left[x+\frac{1}{2}\right] = H_y^{n-\frac{1}{2}}\left[x+\frac{1}{2}\right] + \frac{\Delta t}{\epsilon \Delta x}\left(E_z^n[x+1] - E_z^n[x]\right) \qquad (18)$$

We can use the equations for observe the propagation of the electromagnetic wave inside the medium. For instance, in the popular computational language Python, equations 17 and 18 is written as [18]:

Ez[x] = Ez[x] + (Hy[x] – Hy[x-1]) * imp. (19)

Hy[x] = Hy[x] + (Ez[x+1] – Ez[x])/imp. (20)

In the next section, we use the relations and equations developed until now to observe the propagation of an electromagnetic wave inside a medium.

4. Results and Discussion

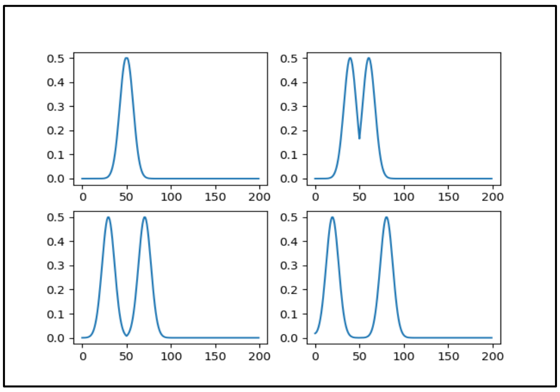

Figure 1: Starting upper left, we see the propagation at some steps of the simulation. We don't have any change in space or time. So, the energy equally travels for both directions.

In **Figure 1** we see an electromagnetic wave propagating inside a isotropic medium with a constant refraction index. Without changes in space or time, propagation occurs smoothly.

What we see until now can deal only with spatial changes. The temporal changes, explored in the second section of this article, ask for some adjustments. Let us briefly review them. In Maxwell's equations, the temporal shifting we deal with is in the functions of permissiveness and permeability. For the sake of simplicity, we will deal with non-magnetic materials. Therefore, our only concern is with the variation of the dielectric constant of the medium. If we have one-dimensional simulation, as above, it is enough to add the terms of the dielectric constant to the electric field equation of updating. The change has the goal to follow such progress in time, in the line of code that deals with the electric field:

Ez[x] = (epsT[i-1]/epsT[i])* Ez[x] + ((Hy[x] – Hy[x-1]) * imp) * epsT[i]. (21)

The variable epsT is responsible for controlling the dielectric constant value over time. In **Figures 2** to **7** we see some images from the previous simulation, with a single change from the first one: the dielectric constant of the medium undergoes a change in a time step of the propagation. We can notice the unusual phenomenon of the temporal reflection of the radiation, due to the change of the medium.

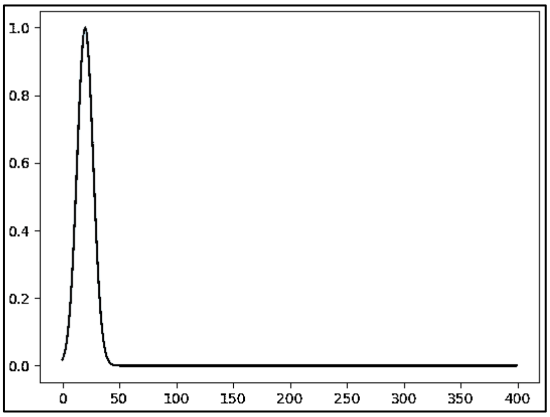

Figure 2: Starting simulation

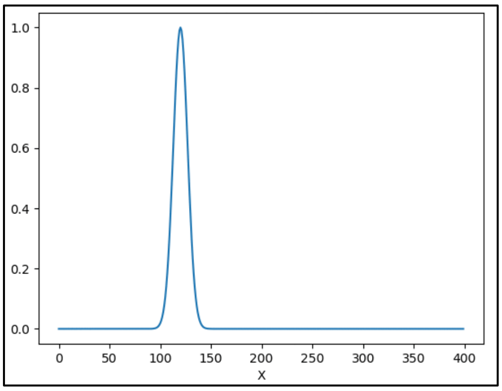

Figure 3: No changes in time or space

For **Figures 2** and **3**, the dielectric constant remains the same since the beginning of the simulation. The power source is at the far left. We see that propagation occurs without losses in this first part of the simulation, as expected. Here, the propagation is like that expected as before.

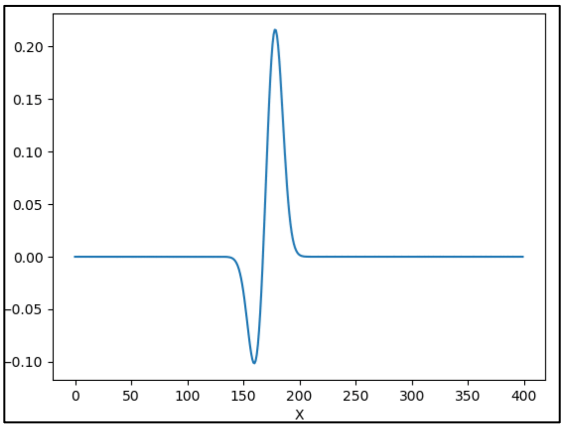

Figure 4: After the temporal change, he reflected wave is evident.

But what does it happen when the dielectric constant changes? Initially, we had $\epsilon = 1$. In this step of the simulation, we have $\epsilon = 9$. However, when the sudden change in the dielectric constant of the medium occurs, the transmitted wave and the reflected wave appears in the space, as shown in **Figure 4**. Of course, unlike the spatial case, where the radiation finds a spatial contour, the energy does not is conserved here. In a non-spontaneous way, the dielectric constant changes, and thus we have an interaction between the medium and the propagating wave.

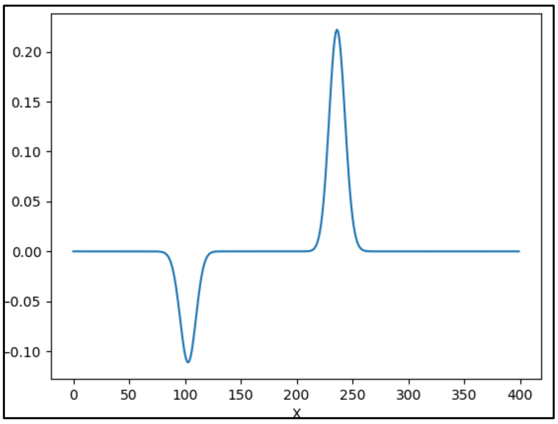

Figure 5: Trivial propagation of the reflected and transmitted waves with new value for dielectric constant.

The **Figure 5** show the trivial propagation of the waves because the dielectric constant remains unaltered in this section of the simulation.

5. Conclusion

We presented an introduction to the propagation of electromagnetic waves in a medium with the dielectric constant (ϵ) varying in time. We highlight the emergence of two temporal phenomena that are already well known when the change in the medium takes place spatially: refraction and reflection. In the case where the change in dielectric constant occurs in time, a reflected wave arises because of the continuity of the fields. The reflected and the incident waves propagates in opposite one another. In this paper, we deal only with temporal changes. But in more general conditions, variations both in time and space can be analyzed. A computational approach was also used. We show how to develop the code for a simulation where the emergence of the phenomena previously described was expected. This demonstration is too advantageous for students who are starting their studies in the area. Often, it is common to misunderstand fundamental steps when a phenomenon is described on a computational basis. In this text, the temporal phenomenon is, for the most part, ignored in the literature. Therefore, inserting it into the simulation can be a significant obstacle for the novice student. However, following the above, the learning curve will be much smoother.

Acknowledgements

We would like to thank the constant support of many colleagues who collaborate with our scientific vision, among them Solange Bessa. We also thank the support of the Instituto Federal de Alagoas and the Universidade Federal de Alagoas.

References

[1]. Mendonça, J. T., and P. K. Shukla., “Time Refraction and Time Reflection: Two Basic Concepts,” Phys. Scr, vol. 65, 2002.

[2]. C. F. Bohren, “Scattering of electromagnetic waves by an optically active cylinder,” J. Colloid Interface Sci., vol. 66, no. 1, pp. 105–109, 1978.

[3]. J. R. Zurita-Sánchez et al., “Pulse propagation through a slab with time-periodic dielectric function ε(t),” Opt. Express, vol. 20, no. 5, p. 5586, 2012.

[4]. J. R. Zurita-Sánchez and P. Halevi, “Resonances in the optical response of a slab with time-periodic dielectric function ε(t),” Phys. Rev. A - At. Mol. Opt. Phys., vol. 81, no. 5, pp. 0–8, 2010.

[5]. J. R. Zurita-Sánchez, P. Halevi, and J. C. Cervantes-González, “Reflection and transmission of a wave incident on a slab with a time-periodic dielectric function (t),” Phys. Rev. A - At. Mol. Opt. Phys., vol. 79, no. 5, pp. 1–13, 2009.

[6]. L. Zeng et al., “Photonic time crystals,” Sci. Rep., vol. 7, no. 1, pp. 1–9, 2017.

[7]. F. R. Morgenthaler, “Velocity Modulation of Electromagnetic Waves,” IEEE Trans. Microw. Theory Tech., vol. 6, no. 2, pp. 167–172, 1958.

[8]. F. Biancalana, A. Amann, A. V Uskov, and E. P. O’Reilly, “Dynamics of light propagation in spatiotemporal dielectric structures,” Phys. Rev. E - Stat. Nonlinear, Soft Matter Phys., vol. 75, no. 4, 2007.

[9]. E. J. Reed, M. Soljačić, and J. D. Joannopoulos, “Reversed doppler effect in photonic crystals,” Phys. Rev. Lett., vol. 91, no. 13, 2003.

[10]. S. Juršenas, S. Miasojedovas, G. Kurilčik, A. Žukauskas, and P. R. Hageman, “Luminescence decay in highly excited GaN grown by hydride vapor-phase epitaxy,” Appl. Phys. Lett., vol. 83, no. 1, pp. 66–

68, 2003.

[11]. E. Lustig, Y. Sharabi, and M. Segev, "Topology of photonic time-crystals."

[12]. E. Lustig, Y. Sharabi, and M. Segev, "Topological aspects of photonic time crystals," 2018.

[13]. J. A. Richards, "Solutions to Periodic Differential Equations," in Analysis of Periodically Time-Varying Systems, Berlin, Heidelberg: Springer Berlin Heidelberg, 1983, pp. 27–49.

[14]. J. Ma and Z.-G. Wang, "Band structure and topological phase transition of photonic time crystals," Opt. Express, vol. 27, no. 9, p. 12914, 2019.

[15]. F. A. Harfoush and A. Taflove, "Scattering of electromagnetic waves by a material half-space with a time-varying conductivity," IEEE Trans. Antennas Propag., vol. 39, no. 7, pp. 898–906, 1991.

[16]. M. N. O. Sadiku, Computational Electromagnetics with MATLAB®. 2018.

[17]. J. Nagel, "The One-Dimensional Finite-Difference Time-Domain (FDTD) Algorithm Applied to the Schrödinger Equation," Bitbucket.Org, vol. 2, no. 3, pp. 1–5, 2012.

[18]. Y. Hao and R. Mittra, FDTD Modeling of Metamaterials, no. January 2008. 2009.