American Scientific Research Journal for Engineering, Technology, and Sciences (ASRJETS)

ISSN (Print) 2313-4410, ISSN (Online) 2313-4402

http://asrjetsjournal.org/

Simulation of Convective Drying with Shrinkage using the Finite Window Method: Application and Validation

Felix Junior Ngouem[a*], Marcel Edoun[b], Louis Monkam[c], Abraham Tetang[d]

[a,b,d]*Université de Ngaoundéré, ENSAI, Laboratoire d'Energétique et de Thermique Appliquée, (LETA)*

[c]*Université de Douala, IUT, Laboratoire de Thermique et Energie (LTE)*

[a]*Email: juniorngouem@gmail.com*

Abstract

This work presents the simulation of drying with shrinkage by the finished window method. To do this, we recalled the drying balance equations and expressed the shrinkage that a product undergoes during the process of product dehydration by means of the shrinkage rate. Then presented the method of resolution employed with an application to the drying of cocoa beans. The different profiles obtained in terms of temperature, water content and volume shrinkage have been shown to be in perfect agreement with the literature. The comparison of the results of the present study with the experimental data of Koua and al., (2019) presents an average relative error of 2.89% for the water content and 0.99% for the reduced volume. The theoretical results are in perfect agreement with the experiments, which gives us a validation criterion of the method proposed as suitable for the resolution of the drying equations.

Keywords: Simulation; convective drying; shrinkage; finite window method.

1. Introduction

Drying is a preservation process that involves reducing the water content of a product so that it can be stored longer. It is a physical phenomenon that simultaneously involves the phenomena of heat and material transfer within the product to be dried as well as between it and the drying medium. This modeling therefore requires the writing of a complex system of partial differential equations (PDE) coupled and non-linear for the most part and whose resolution can only be done by so-called numerical methods. During the drying of products with a high-water content, there is usually a shrinkage phenomenon.

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* Corresponding author.

This is a phenomenon which reduces the volume of the sample and affects the exchange surface between the product to be dried and the drying medium [1]. Shrinkage is the reduction in the volume of the product sample due to the departure of the water contained therein [2]. Drying with shrinkage has been studied by many authors. Reference [3-4] model the rate of shrinkage of the air-product surface during drying using the Leibniz-Reynold transport theorem (TTLR). Reference [5] propose models for the mass diffusion coefficient and the rate of withdrawal and determine the parameters of these by comparing their results to the experiment. [6] show that the moisture content decreases more rapidly in the drying model with shrinkage, which leads to the conclusion that this one gives shorter drying times. Reference [7] show that neglecting the effect of shrinkage can lead to false results. Reference [8] postulate a local shrinkage rate proportional to the local mass diffusion flux, the coefficient of proportionality called the shrinkage factor. This model applied to 2D potato slice drying gives good results in terms of predicting volume reduction, surface deformation and effective diffusivity of water. A synthesis of the main models developed as well as the resolution methods used are presented [9-11]. From the above, we can easily see the influence of shrinkage on the drying kinetics. Unfortunately, the shrinkage phenomenon is not systematically taken into account in drying models, and particularly with regard to work on cocoa. The objective of this work is to present a new approach to solving the drying equations and to apply it to the case of cocoa drying taking into account shrinkage. To do this we will:

- Presented the drying equations with shrinkage;

- Presented the digital method of resolution;

- Apply the method presented to the case of cocoa.

2. Mathematical model

2.1. Balance equations

Convective drying with shrinkage is defined by the following balance equations:

Mass balance:

$$\frac{\partial c}{\partial t} + \vec{\nabla}\left(\vec{u}c\right) = \vec{\nabla}\left(\overrightarrow{D\nabla c}\right) \quad (1)$$

Heat balance:

$$\left(\rho C_P\right)_s \left[\frac{\partial T}{\partial t} + \vec{\nabla}\left(\vec{u}\cdot T\right)\right] = \vec{\nabla}\left(\overrightarrow{k\nabla T}\right) \quad (2)$$

air-product interface:

$$-D\nabla c\big|_{x\in s(t)} = h_m\left(c\big|_{s(t)} - c_{air}\right) \quad (3)$$

$$-k\nabla T\big|_{x\in s(t)} = h\left(T_a - T\big|_{s(t)}\right) - h_m\left(c\big|_{s(t)} - c_a\right)L_v \quad (4)$$

2.2. Expression of shrinkage

The shrinkage is expressed through the shrinkage speed by [8]:

$$v(x) = -\alpha(\phi)\,J(x) \qquad (5)$$

The coefficient α is called the shrinkage factor and is determined from the experimental drying results. As for the displacement of the air-product surface, it is given by:

$$\frac{d\,x\big|_{\in s(t)}}{dt} = v = \alpha(\phi)\,D\nabla\phi\big|_{x\in s(t)} \quad (6)$$

3. Numerical resolution

3.1. Method presentation

This work is based on a new numerical method for solving partial differential equations (PDE), in particular the drying equations. This method, called the “finite window method” (MFF), was developed by [13] and used for the resolution of thermal problems [14-15]. In the finite window method, a continuous approximation of the desired function is required in the domain knowing the values of said function at the nodes:

$$\tilde{f} = \sum_i N_i(x,y,z)\,f_i(t) \qquad (7)$$

The functions $N_i(x,y,z)$ represent the interpolation functions and *n* the number of nodes contained in a Finite Window (FF) and $f_i(t)$ the value of the unknown function at node *i*. By considering an unspecified node k in the field, one formulates its finite window, then one introduces the approximated function in the equation (with partial or ordinary derivatives) to be solved, one thus obtains an error due to the approximation of the sought function and noted δ_k one searches for the values of $f_i(t)$ which minimizes this error by applying the criterion of the least squares in the corresponding FF either:

$$er_k = \int_{ff} \delta_k^2\,df \qquad (8)$$

By applying to all the nodes of the study domain, we finally obtain the global error in the whole domain:

$$R = \sum_{k\in\Omega}\int_{ff} \delta_k^2\,df \qquad (9)$$

Therefore, the values of f which minimizes the overall error in each node if they exist must verify the following condition:

$$dR = 0 \qquad (10)$$

Avec

$$dR = \sum_{k\in\Omega} 2\int_{ff} d\delta_k \cdot \delta_k df \qquad (11)$$

So finally, dR gets the following form:

$$dR = 2\sum_{k\in\Omega}\left[\sum_i \sum_j a_{ij} u_j\right] du_i \qquad (12)$$

Now it suffices to pose $dR = 0$ to find the value of u_j :

$$\sum_{k\in\Omega}\left[\sum_i \sum_j a_{ij} u_j\right] = 0 \qquad (13)$$

with $u_i = f_i(t)$

For each node k and as a result of the expansion of eq (13), we end up with the system eq. (14) or the coefficients a_{ij}^{f} are evaluated in the *k* window and u_j^{f} the values of the function at all the nodes j belonging to the zone of influence of k (window of k), including in *k* itself.

$$\begin{cases} a_{11}^f u_1^f + a_{12}^f u_2^f + \cdots + a_{1n}^f u_n^f = 0 \\ a_{21}^f u_1^f + a_{22}^f u_2^f + \cdots + a_{2n}^f u_n^f = 0 \\ \vdots \\ a_{n1}^f u_1^f + a_{n2}^f u_2^f + \cdots + a_{nn}^f u_n^f = 0 \end{cases} \qquad (14)$$

The previous system can be in the following matrix form:

$$A_k^f \cdot U_k^f = 0 \qquad (15)$$

By repeating the process for each node of the domain, we construct the matrix system (Eq. 16) whose resolution allows to have the values of the function sought at each point of the domain through the vector U:

$$AU = B \tag{16}$$

Matrix A is built from small matrices A_k^f defined on each window and the column vector B is built from the boundary conditions

4. Application

The different residues are expressed by:

4.1. Mass equation

- Internal node

$$\delta_k = \sum_i \xi_i X^n - \sum_i \tilde{N}_i X^{n-1} \tag{17}$$

avec

$$\xi_i = s\varepsilon\left[sN + \varepsilon\Delta s^n \frac{dN}{d\varepsilon}\right] - \Delta t\bar{D}\left[2\frac{dN}{d\varepsilon} + \varepsilon\frac{d^2N}{d\varepsilon^2}\right]$$

$$\tilde{N}_i = s^2\varepsilon N$$

- Surface node

$$\delta_k = \sum_i dN_i X^n - sBi_m\phi_a \tag{18}$$

- Center node

$$\delta_k = \frac{dN}{d\varepsilon} X^n \tag{19}$$

4.2. Heat equation

- Internal node

$$\delta_k = \sum_i \varphi_i T^n - \sum_i \tilde{N}_i T^{n-1} \tag{20}$$

$$\varphi_i = s\varepsilon\left[sN + \varepsilon\Delta s^n \frac{dN}{d\varepsilon}\right] - \Delta tLe\left[2\frac{dN}{d\varepsilon} + \varepsilon\frac{d^2N}{d\varepsilon^2}\right]$$

- Surface node

$$\delta_k = \sum_i d\tilde{N}_i T^n + s\beta \sum_i NX^n - s\left(Bi + \beta\phi_a\right) \qquad (21)$$

- Center node

$$\delta_k = \frac{dN}{d\varepsilon} T^n \qquad (22)$$

The optimization consists in determining for each node k of the domain (mesh) the values of the unknown function (temperature and water content) which minimizes the overall error. For this its differential is set equal to 0, that is:

- reduced moisture:

$$CX^n - DX^{n-1} + HX^n - sBi_m\phi_a K + UX^n = 0 \quad (23)$$

$$X^n = y \backslash \left[DX^{n-1} + sBi_m\phi_a K \right]$$

with $y = C + H + U$

- reduced temperature:

$$GT^n - NT^{n-1} + PT^n + s^n\beta QX^n - s^n\left(Bi + \beta\phi_a\right)R + UT^n = 0 \qquad (24)$$

$$T^n = w \backslash \left[N \cdot T^{n-1} - s^n\beta Q \cdot X^n + s^n\left(Bi + \beta\phi_a\right) \cdot R \right]$$

With $w = G + P + U$

- volume shrinkage $s^n = s^{n-1} - \Delta t \Delta s^n$

$$\Delta s^n = \alpha \frac{\rho_s}{\rho_e} X_0 Bi_m \left(\phi_s^n - \phi_a \right) \qquad (25)$$

X^n, T^n et s^n represent the reduced moisture content, the reduced temperature and the volume shrinkage at each instant n. Matrices C, D, H, K, U, G, N, P, Q, et R are built from the different residues.

5. Results

The following results were obtained after simulation of the previously established equations for the case of cocoa bean drying. The shrinkage factor used in the present study is derived from the analysis of indirect drying data from [12] from the algorithm developed by Adrover and his colleagues [8]. It is 0.396979, a value that will be used throughout this study.

6. Validation

In the vast majority of numerical studies of agricultural product drying, validation of results is done by comparing theoretical and experimental results, and the mean relative error is the magnitude generally used in the validation process. The comparison of the results of this study with the experimental data of the indirect drying of cocoa beans from [12] is presented in figures 1 and 2. We observe a very good match between the theoretical and experimental results. The results are validated by calculating the average relative error between experimental and theoretical values (eq. 26). For the case of the moisture content and the reduced volume we obtained mean relative errors of 2.89% and 0.99%. Considering that this is generally greater than or equal to 3%, the values presented in this study being significantly lower than this limit of 3%, we can, in the light of our results, conclude that the numerical method used combined with the phenomenon of shrinkage makes it possible to obtain more precise results.

$$E(\%) = \sum_{i=1}^{n} \frac{\left| X_{\exp} - \langle X_{sim} \rangle \right|}{X_{\exp}} \frac{100}{n} \qquad (26)$$

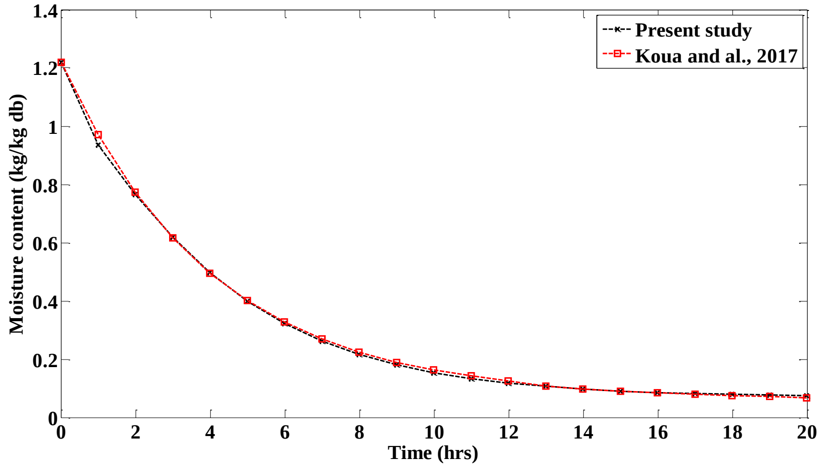

Figure 1: Comparison of the moisture content kinetics of the present study with the experimental results of Koua and al., [12]

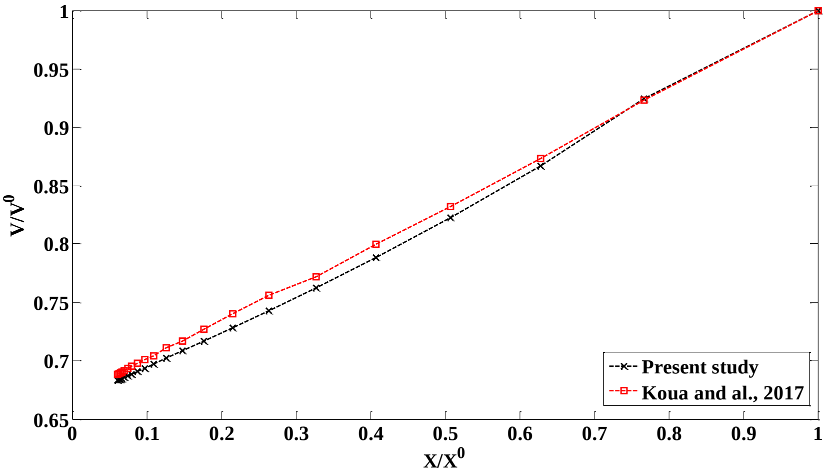

Figure 2: Comparison of the evolution of the reduced volume the present study with the experimental results of Koua and al., [12]

Figures 3 and 4 show the time profiles of water content and temperature as a function of thickness. It can be seen that for the temperature, there is practically no variation along the radius, which is certainly due to the low thermal conductivity of cocoa. It therefore evolves only over time. The moisture in the product moves from the center to the surface and over time this tends to become uniform throughout the bean. Reference [2] present a similar profile for the case of apple drying taking into account the shrinkage for a constant diffusion coefficient with a finite difference numerical scheme. The similarity of the profiles obtained for different products under similar conditions sufficiently proves the capacity of the method presented here to describe the drying processes. Figure 5 shows the kinetics of reduced moisture content. This is evaluated at the center and on the surface of the bean in addition to its average value. The moisture content drops exponentially. This decrease is faster on the surface than at the center. We see that drying begins from the first moments of the process. During the heat-up phase, water transfer occurs both at the center and at the surface of the product. Therefore, the water transport is not influenced by the temperature gradient, but only by the concentration gradient. Taking into account the difficulty in experimentally evaluating the water content during the dehydration of the products, one often has recourse to semi-empirical models. The average profile of reduced water content presented here perfectly reproduces the evolution of the average moisture content reported by [16] for the artificial drying of cocoa beans. As the semi-empirical models do not take into account the shrinkage, we can say that for the cases of cocoa beans, the model presented is well suited to the study of kinetics.

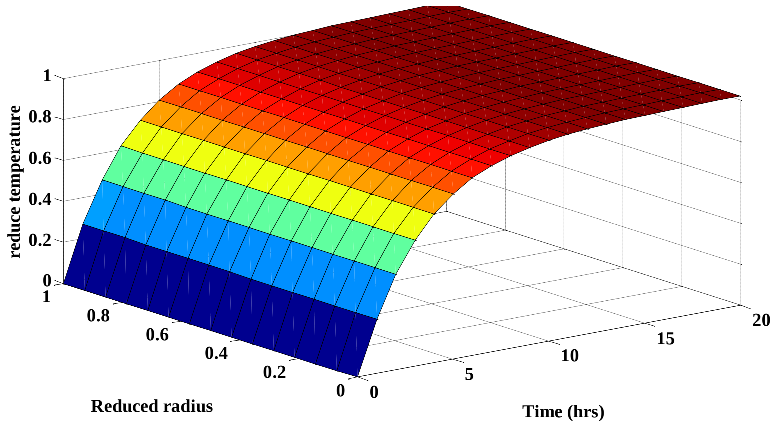

Figure 3: 3D profile of reduced temperature

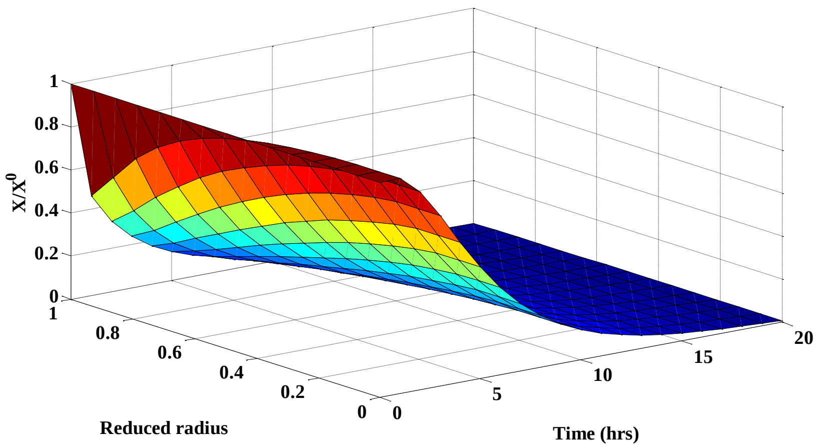

Figure 4: 3D profile of reduced moisture content

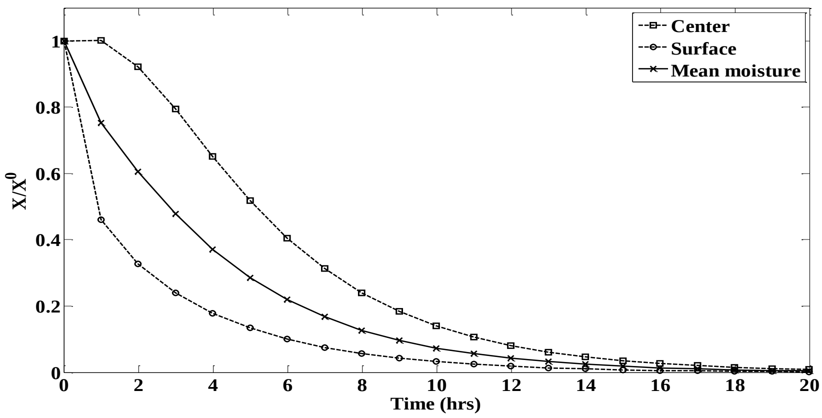

Figure 5: Kinetics of water loss and temperature in the bean during drying from the surface to the center.

Figure 6 shows the reduced volume shrinkage as a function of time. In this study, the local shrinkage velocity is set proportional to the local mass flow, this rate also being that of volume variation during the dehydration process, therefore as long as there is a gradient of content in the bean, the volume will decrease proportionally until the gradient vanishes. The profile obtained will therefore be identical to that of water content. [17] by studying the couplings between mass transfer, heat transfer and shrinkage obtained identical results for the evolution of the volume shrinkage of the product. However, to achieve this they develop a complete model of microscopic porous medium type by considering the different phases constituting the product. The added value of this study lies in the ability of our model to predict the volume shrinkage of the product despite its simplicity, just like a model based on more detailed modeling of transfers inside the product and in particular its deformation. such as that [17].

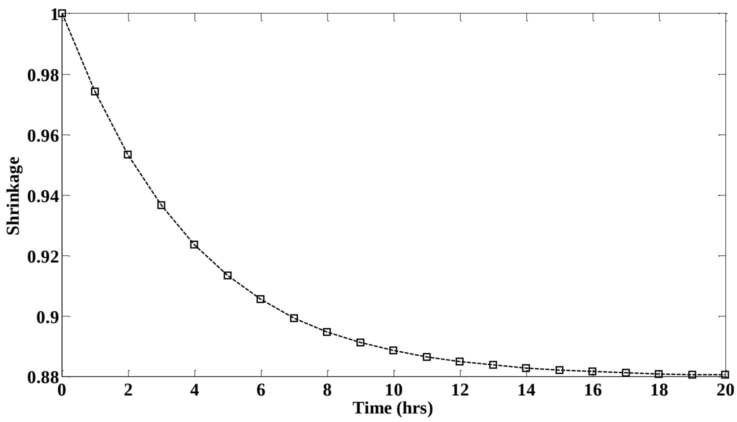

Figure 6: Variation of volume shrinkage during drying.

7. Conclusion

The objective of this study was to simulate the convective drying equations taking into account the shrinkage phenomenon by a new numerical approach to solving PDEs, namely the finite window method. The results obtained were validated by comparison of these with the experimental results presented by [12]. These results present an average relative error of 2.89% for the water content and 0.99% for the volume shrinkage. Likewise, the analysis of the different profiles shows that they are in perfect agreement with those presented in the literature, in particular with the work of [2,16]. The results also show that taking into account the shrinkage phenomenon allows better results to be obtained, therefore it should be systematically integrated into drying models. The present study finally allows us to validate our numerical method of resolution of the PDEs, in particular the drying equations. The main limitation of this study remains the evaluation of the shrinkage factor, which is highly dependent on the operating conditions and therefore subject to errors in its determination and therefore likely to affect the results.

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