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Stability Validation: Righting Moment, Density & List Angle of Floating Objects

Mohamed Abdalla Almheriegh*

Associate Professor, Department of civil Engineering, Faculty of Engineering - Tripoli University, P O Box 82677, Tripoli, Libya

Email: malmherigh@gmail.com

Abstract

Finite Element modelling of floating structure for the sake of stress analysis has never been an easy task due to various numbers of factors involved in such structures and the nature of finite element simulations. Typically, if using ‘ALE’ (Arbitrary Lagrangian Eulerian) feature available in LS-DYNA3D explicit code, the floating object will have to float in a fluid 'half-space' of a size which will best represent the effects of the pseudo-infinite sea. The boundaries will need careful consideration, and perhaps employ non-reflecting (energy absorbing) representation. In this representation which will be employed in this paper a study of buoyancy is conducted for verification purposes for modelling afloat. Discussed in this paper is the mathematical verification of the buoyancy of a float, while modelling with LS-DYNA3D features is left to further consideration.

Keywords: advanced formulation of floating object; ALE features; finite element simulation of float; load modelling of floating structure.

1 Introduction

One of the main problems in structural analysis of a floating object is the assurance of stability necessary for the floating structure to function the purpose it is designed for**.** Both good knowledge of buoyancy mathematics and well-established understanding are discussed and employed to strongly comprehend the action of the structure in order to serve design purpose. A small float 2-dimentional cube representing model is assumed and validated, the ALE feature employed by explicit analysis codes is discussed. The list angle of stability is measured to insure stability. Other feature of floating object stability is righting or restoring moment as a measure for stability is also discussed and calculated. Mathcad sheet is created to solve buoyancy equations, a graphical presentation and tabular values measuring stability are introduced. Broad discussion of previous work is given in References [1,2,3,5] with wide number of related references are also reported. Study of stability of floating bodies is a conventional subject in fluid mechanics.

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* Corresponding author.

2. Validation of Buoyancy

To confirm the ability of modelling the effects of buoyancy correctly, first; a simple two-dimensional representation of a cuboid float is examined using MathCAD calculation, hence preparing for further validation using the LS-DYNA3D explicit code 'ALE' feature, References [1] through [4]. To choose geometry and density of a floating structure, a floating block of different densities and heights is mathematically investigated. The dimensions of the float examined are shown in the following Figure (1), the mathematical investigation involves change of height, and density, then calculating the angle of list which is the measure of stability.

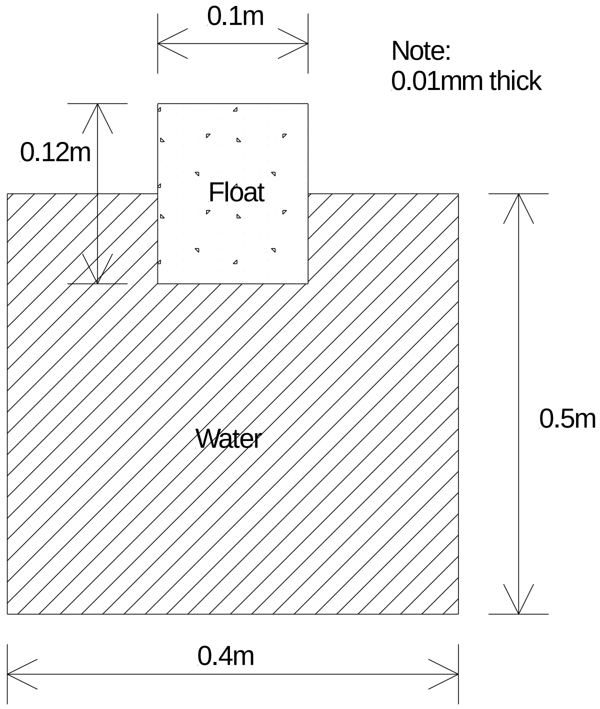


Figure 1: Two-dimensional representation of float model

There are a number of parameters which can be studied in terms of geometry and material properties, but for this exercise, only the influence of float density was studied. A number of different cases were chosen to illustrate different cases of floating stability and the angles of list expected to be taken up by the float. Firstly, A discussion regarding the theory of buoyancy is presented in next heading.

3. Buoyancy and Stability of Floating Objects

One of the most important problems involving buoyancy is the determination of the stability of a floating object. The analysis may be illustrated by considering a body shown in a cross section in an upright position Figure (2 a). Point B is the centroid of the displaced volume and is known as the centre of buoyancy. The resultant of the forces exerted on the body by the water pressure is the force F_B. Force F_B passes through B and is equal and opposite to body weight W.

If the body is caused to list through an angle $\Delta\theta$, Figure (2 b) the shape of the displaced volume changes and the centre of buoyancy will shift to some new position such as B'. The point of intersection of the vertical line through B' with the centreline of the unlisted body is called the metacentre M and the distance measuring how far M from the centre of mass G is known as metacentric height. For most hull shapes the metacentric height remains practically constant for angles of list up to about 20^{o}. When M is above G, as in Figure (2 b), there is clearly a righting moment which tends to bring the body back to its original position. The magnitude of this moment for any particular angle of list is a measure of the stability of the body. If M is below G, as for the body of Figure (2 c), the moment accompanying any list is in the direction to increase the list. This is clearly a condition of instability and strictly avoided in the design of any floating object. This methodology has been observed in the models to be presented in the next headings. For a floating object where the weight and horizontal forces act so far above the centre of buoyancy, significant ballast must be added to the lowest possible location, thus the centre of gravity is moved down to increase stability. The larger the metacentric height 'GM' the greater is the restoring (righting) moment.

A floating body is stable if, when it is displaced, it returns to equilibrium.

A floating body is unstable if, when it is displaced, it moves to a new equilibrium.

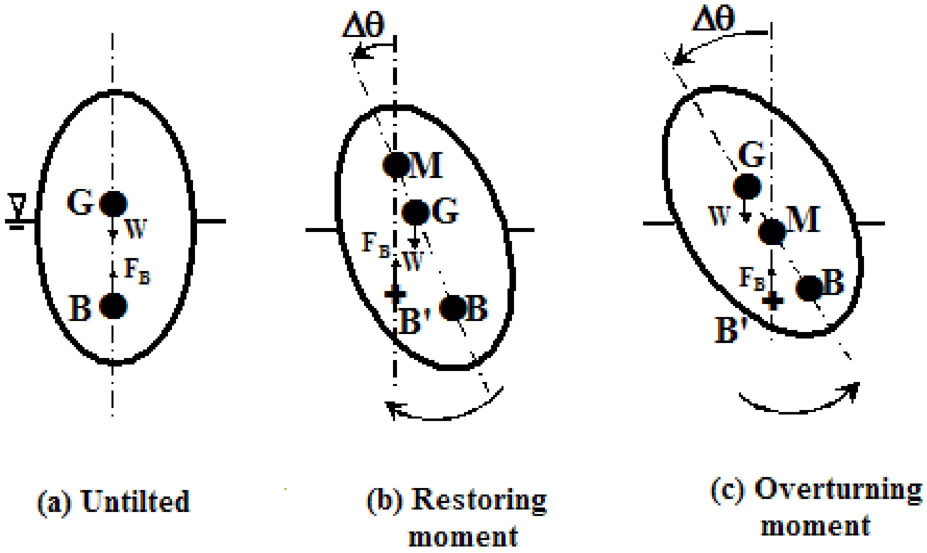


Figure 2: Stability of floating bodies

In the next shown illustration, stability is attained if the metacentric height, MG, is positive (MG = $z_M - z_G > 0$). If the metacenter, M, lies below the center of gravity, G, then the body is unstable. In other words the metacentric height, MG, is negative (MG < 0).

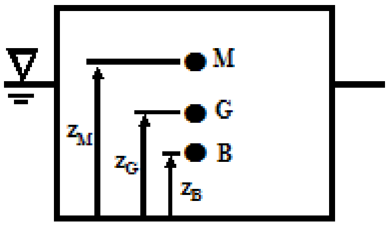


Figure 3

Where metacentric height, MG, is given by:

$$MG = MB - GB \quad \text{or} \quad MG = \frac{I}{V_S} - GB$$

Where I is the 2nd moment of area (moment of inertia) of the plane section of the body where it cuts the waterline. In other words, if someone were to cut horizontally through the body at the water surface and look at the area of the body exposed by the cut, I is the 2nd moment of area of that body about the longest axis. While V_S is the submerged volume (i.e. volume of fluid displaced) and GB is the distance between the center of gravity and the center of buoyancy ($GB = z_G - z_B$) found from the geometry.

To conclude; steps for solving buoyancy problems are:

(1) From geometry of body and density of fluid and body equate; Weight of displaced fluid = Total weight of body. This gives the depth of immersion of the body or the weight (density) of the body, whichever is unknown.

(2) To assess stability, first find the location of the center of gravity G of the body.

(3) Then, find the location of the center of buoyancy B (centroid of displaced volume). For a regularly shaped body (cuboid) this will be at half the height of the immersed portion of the body.

(4) Calculate the distance GB.

(5) Calculate MB, using $MB = I / V_S$, ($I = \pi D^4/64$ for a circular section body and $bd^3/12$ for a rectangular section body, D is diameter, b and d are the sides of the rectangle).

(6) Calculate metacentric height, ($MG = z_M - z_G$), from $MG = MB - GB$. If $MG > 0$ then body is stable. If $MG < 0$ then body is unstable while $MG = 0$ is neutral stability.

In the proceeding MathCAD templates, Mohamed, [5], the theoretical behaviour of the buoyancy action of the above mentioned 2-D model of varying float density is illustrated. The buoyancy stability is achieved at different listing angles for the various densities. These sheets are as follows:

Buoyancy Stability Check - Case No 1

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1 \cdot m$ — Depth $D := 0.12 \cdot m$

Thickness $T := 0.01 \cdot m$

Density of object $\rho_s := 87.98\,kg \cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg \cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s \cdot B \cdot D \cdot T}{\rho_w \cdot B \cdot T} \qquad h = 0.01m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2 \cdot \frac{\rho_s \cdot D}{\rho_w} - B \cdot \tan(\theta)}{2} \qquad d_2(\theta) := B \cdot \tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta) \cdot B \cdot \frac{B}{2}\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{3} \cdot B}{\left(d_1(\theta) \cdot B\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B \cdot \frac{d_1(\theta)^2}{2} + B \cdot \frac{d_2(\theta) - d_1(\theta)}{2} \cdot \left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta) \cdot B\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1 \cdot deg, 2 \cdot deg .. 37 \cdot deg$

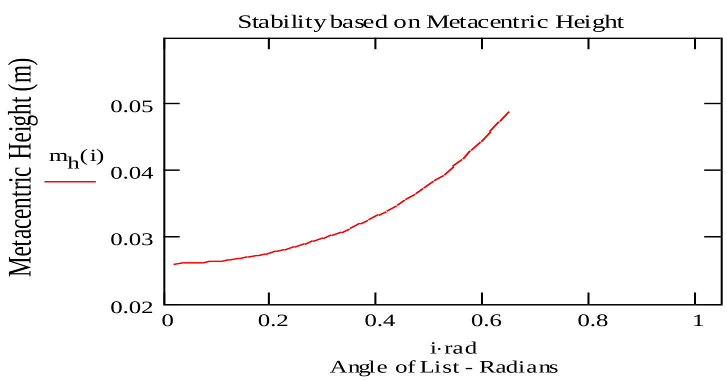
$m_h(0.01 \cdot deg) = 26.055mm$

STABILITY := if($m_h(0.01 \cdot deg) > 0$, "STABLE" , "UNSTABLE")

STABILITY = "STABLE"

Angle of List

THETA = 0 degrees



Buoyancy Stability Check - Case No 2

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 113.6\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.013m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

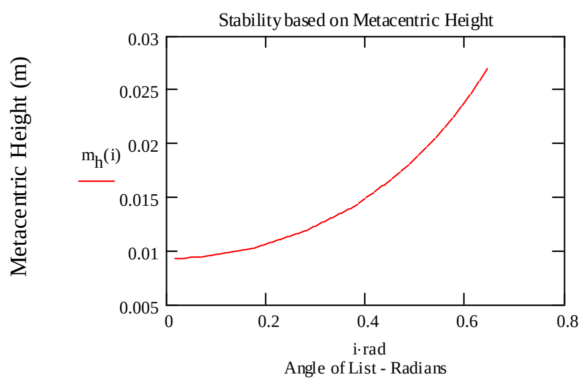
$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1\cdot deg, 2\cdot deg .. 37\cdot deg$ $\qquad m_h(0.01\cdot deg) = 9.309mm$

STABILITY := if($m_h(0.01\cdot deg) > 0$,"STABLE" ,"UNSTABLE") $\qquad$ STABILITY = "STABLE"

Angle of List

THETA = 0 degrees



Buoyancy Stability Check - Case No 3

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 136.9\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.016m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\dfrac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\dfrac{B}{2}\right) + \dfrac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \dfrac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\dfrac{d_1(\theta)^2}{2} + B\cdot\dfrac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \dfrac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \dfrac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

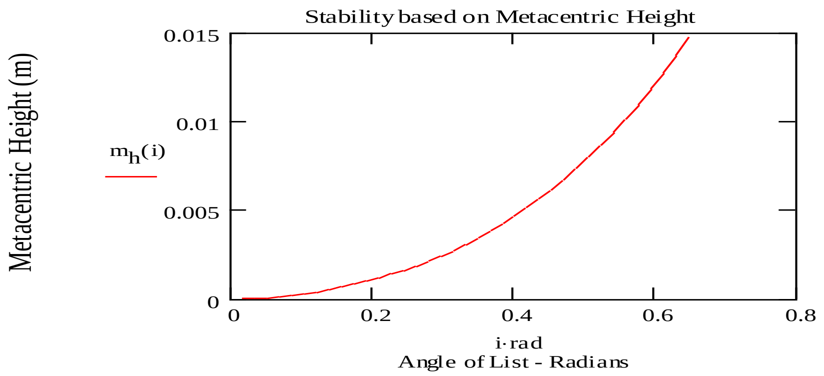
$$m_h(\theta) := \frac{x_{bar}(\theta) - \dfrac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1\cdot deg, 2\cdot deg \,..\, 37\cdot deg$ $\qquad m_h(0.01\cdot deg) = 0.008mm$

$STABILITY := if\left(m_h(0.01\cdot deg) > 0, \text{"STABLE"}, \text{"UNSTABLE"}\right) \qquad STABILITY = \text{"STABLE"}$

Angle of List

THETA = 0 degrees — AT CHANGEOVER FROM VERTICAL TO LISTING STABILITY



Buoyancy Stability Check - Case No 4

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 228.07\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.027m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

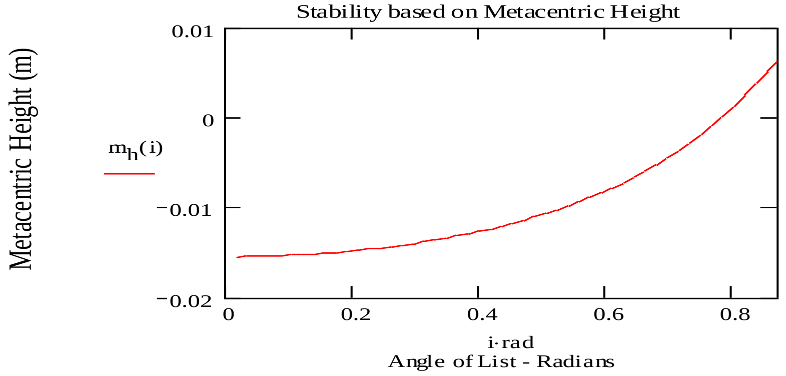
$i := 1\cdot deg, 2\cdot deg\,..\,50\cdot deg$

$m_h(0.01\cdot deg) = -15.44mm$

STABILITY := if($m_h(0.01\cdot deg) > 0$, "STABLE", "UNSTABLE") STABILITY = "UNSTABLE"

Angle of List

Stability achieved at 45 deg $m_h(44.9\cdot deg) = 0.057mm$



Buoyancy Stability Check - Case No 5

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 341.67\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.04m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1\cdot deg, 2\cdot deg .. 60\cdot deg$

$m_h(0.01\cdot deg) = -19.167mm$

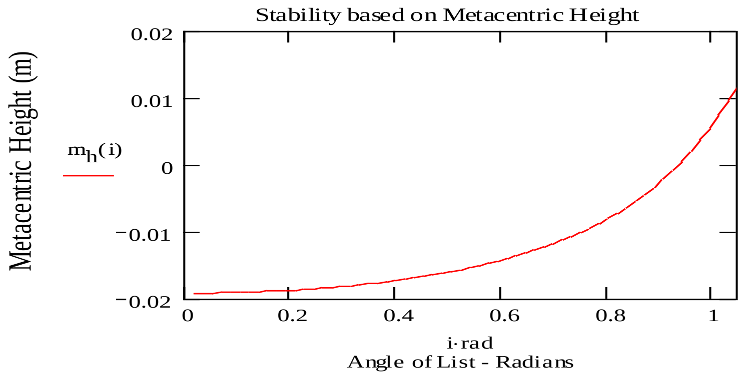
STABILITY := if($m_h(0.01\cdot deg) > 0$, "STABLE" , "UNSTABLE")

STABILITY = "UNSTABLE"

Angle of List

Stability achieved at 54 deg

$m_h(53.7\cdot deg) = 0.138mm$



Buoyancy Stability Check - Case No 6

$kN := 1000\,N$

Problem Data

Size of object Breadth $B := 0.1\cdot m$ Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 455.271\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.053m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

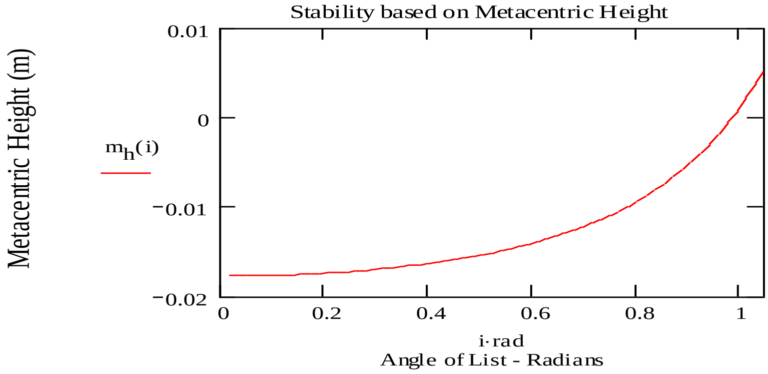
$i := 1\cdot deg, 2\cdot deg\,..\,60\cdot deg$

$m_h(0.01\cdot deg) = -17.715mm$

STABILITY := if($m_h(0.01\cdot deg) > 0$, "STABLE" , "UNSTABLE") STABILITY = "UNSTABLE"

Angle of List

Stability achieved at 56.5 deg $m_h(56.5\cdot deg) = 0.129mm$



Buoyancy Stability Check - Case No 7

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 569.729\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.067m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

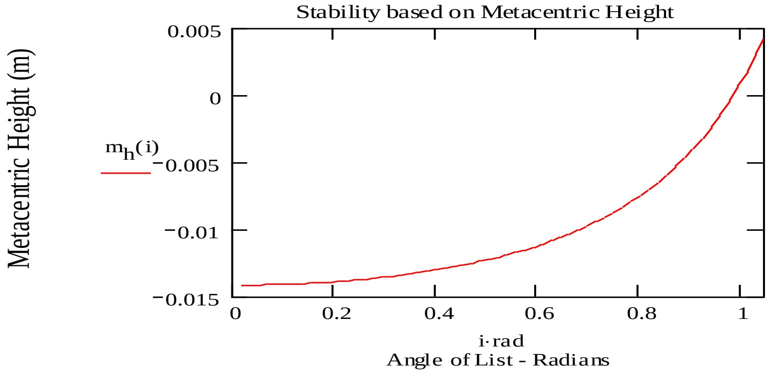
$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1\cdot deg, 2\cdot deg .. 60\cdot deg$ $\qquad m_h(0.01\cdot deg) = -14.156mm$

STABILITY := if$\left(m_h(0.01\cdot deg) > 0, \text{"STABLE"}, \text{"UNSTABLE"}\right)$ $\qquad$ STABILITY = "UNSTABLE"

Angle of List

Stability achieved at 56.5 deg $\qquad m_h(56.5\cdot deg) = 0.103mm$



Buoyancy Stability Check - Case No 8

$kN := 1000\ N$

Problem Data

Size of object — Breadth $B := 0.1 \cdot m$ — Depth $D := 0.12 \cdot m$

Thickness $T := 0.01 \cdot m$

Density of object $\rho_s := 683.333 kg \cdot m^{-3}$

Density of Water $\rho_w := 1025 kg \cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s \cdot B \cdot D \cdot T}{\rho_w \cdot B \cdot T} \qquad h = 0.08m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2 \cdot \frac{\rho_s \cdot D}{\rho_w} - B \cdot \tan(\theta)}{2} \qquad d_2(\theta) := B \cdot \tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta) \cdot B \cdot \frac{B}{2}\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{3} \cdot B}{\left(d_1(\theta) \cdot B\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B \cdot \frac{d_1(\theta)^2}{2} + B \cdot \frac{d_2(\theta) - d_1(\theta)}{2} \cdot \left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta) \cdot B\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

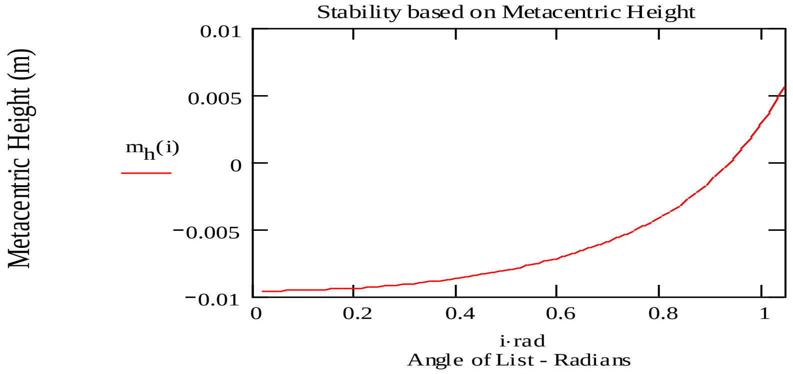
$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1 \cdot deg, 2 \cdot deg .. 60 \cdot deg$ $\qquad m_h(0.01 \cdot deg) = -9.583mm$

$STABILITY := if\left(m_h(0.01 \cdot deg) > 0, \text{"STABLE"}, \text{"UNSTABLE"}\right)$ $\qquad STABILITY = \text{"UNSTABLE"}$

Angle of List

Stability achieved at 53.7 deg $\qquad m_h(53.7 \cdot deg) = 0.069mm$



Buoyancy Stability Check - Case No 9

$kN := 1000\ N$

Problem Data

Size of object — Breadth $B := 0.1 \cdot m$ — Depth $D := 0.12 \cdot m$

Thickness $T := 0.01 \cdot m$

Density of object $\rho_s := 796.938 kg \cdot m^{-3}$

Density of Water $\rho_w := 1025 kg \cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s \cdot B \cdot D \cdot T}{\rho_w \cdot B \cdot T} \qquad h = 0.093m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2 \cdot \frac{\rho_s \cdot D}{\rho_w} - B \cdot \tan(\theta)}{2} \qquad d_2(\theta) := B \cdot \tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta) \cdot B \cdot \frac{B}{2}\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{3} \cdot B}{\left(d_1(\theta) \cdot B\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B \cdot \frac{d_1(\theta)^2}{2} + B \cdot \frac{d_2(\theta) - d_1(\theta)}{2} \cdot \left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta) \cdot B\right) + \frac{B \cdot \left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1 \cdot deg, 2 \cdot deg .. 60 \cdot deg$

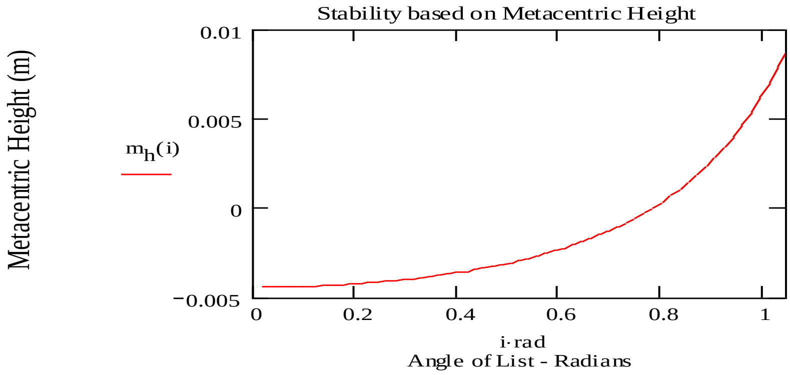
$m_h(0.01 \cdot deg) = -4.418mm$

STABILITY $:= \text{if}\left(m_h(0.01 \cdot deg) > 0, \text{"STABLE"}, \text{"UNSTABLE"}\right)$

STABILITY = "UNSTABLE"

Angle of List

Stability achieved at 44.9 deg $\qquad m_h(44.9 \cdot deg) = 0.017mm$



Buoyancy Stability Check - Case No 10

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 854.17\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.1m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

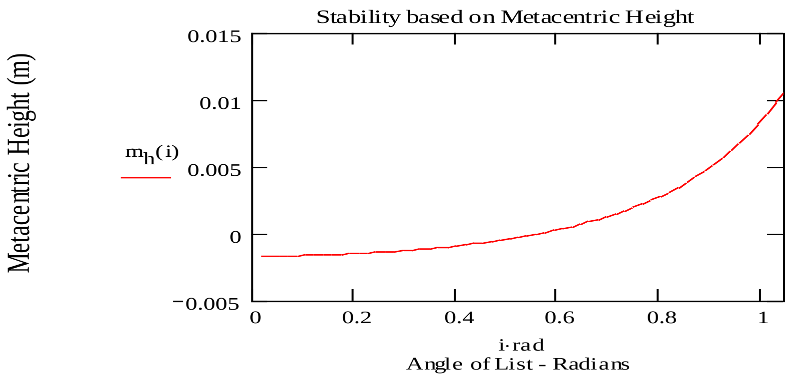
$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1\cdot deg, 2\cdot deg\,..\,60\cdot deg$ $\qquad m_h(0.01\cdot deg) = -1.667mm$

STABILITY $:= \text{if}\left(m_h(0.01\cdot deg) > 0, \text{"STABLE"}, \text{"UNSTABLE"}\right)$ $\qquad$ STABILITY = "UNSTABLE"

Angle of List

Stability achieved at 32.4 deg $\qquad m_h(32.4\,deg) = 0.012mm$



Buoyancy Stability Check - Case No 11

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 888.1\cdot kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.104m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta) - d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta) - d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1\cdot deg, 2\cdot deg .. 60\cdot deg$

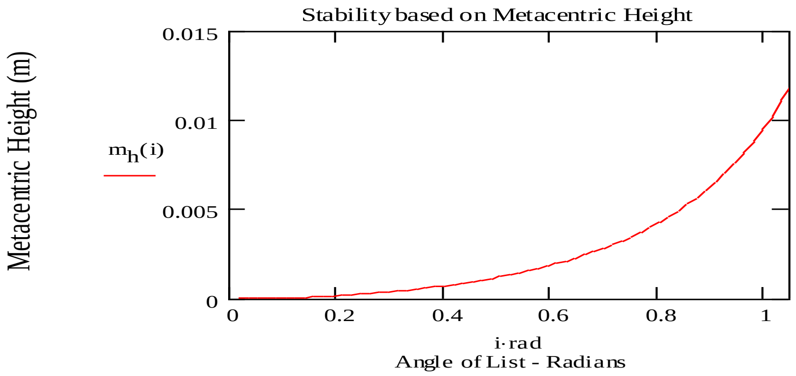
$m_h(0.01\cdot deg) = 0.001mm$

STABILITY := if($m_h(0.01\cdot deg) > 0$, "STABLE" , "UNSTABLE") STABILITY = "STABLE"

Angle of List

THETA = 0 degrees

AT CHANGEOVER FROM
LISTING TO VERTICAL STABILITY



Buoyancy Stability Check - Case No 12

$kN := 1000\,N$

Problem Data

Size of object — Breadth $B := 0.1\cdot m$ — Depth $D := 0.12\cdot m$

Thickness $T := 0.01\cdot m$

Density of object $\rho_s := 900\,kg\cdot m^{-3}$

Density of Water $\rho_w := 1025\,kg\cdot m^{-3}$

Displaced height of water

$$h := \frac{\rho_s\cdot B\cdot D\cdot T}{\rho_w\cdot B\cdot T} \qquad h = 0.105m$$

Geometry of Part Submerged Bouyant Object

$$d_1(\theta) := \frac{2\cdot\frac{\rho_s\cdot D}{\rho_w} - B\cdot\tan(\theta)}{2} \qquad d_2(\theta) := B\cdot\tan(\theta) + d_1(\theta)$$

Position of centroid

$$x_{bar}(\theta) := \frac{\left(d_1(\theta)\cdot B\cdot\frac{B}{2}\right) + \frac{B\cdot\left(d_2(\theta) - d_1(\theta)\right)}{3}\cdot B}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta)-d_1(\theta)\right)}{2}}$$

$$y_{bar}(\theta) := \frac{B\cdot\frac{d_1(\theta)^2}{2} + B\cdot\frac{d_2(\theta)-d_1(\theta)}{2}\cdot\left(d_1(\theta) + \frac{d_2(\theta)-d_1(\theta)}{3}\right)}{\left(d_1(\theta)\cdot B\right) + \frac{B\cdot\left(d_2(\theta)-d_1(\theta)\right)}{2}}$$

Metacentre height above Centre of gravity (represented on subsequent graph plot)

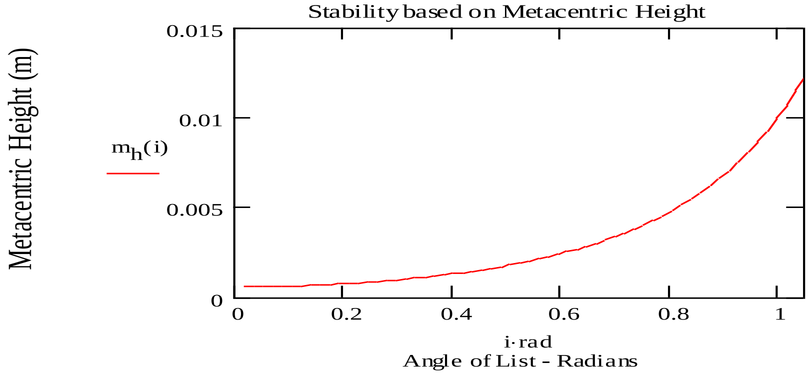
$$m_h(\theta) := \frac{x_{bar}(\theta) - \frac{B}{2}}{\tan(\theta)} - \left(\frac{D}{2} - y_{bar}(\theta)\right)$$

$i := 1\cdot deg\,, 2\cdot deg\,..\,60\cdot deg$ $\qquad m_h(0.01\cdot deg) = 0.592mm$

STABILITY $:= \text{if}\left(m_h(0.01\cdot deg) > 0, \text{"STABLE"}, \text{"UNSTABLE"}\right)$ $\qquad$ STABILITY = "STABLE"

Angle of List

THETA = 0 degrees



4 Conclusions

The preceding calculations address the theoretical mechanics of stability of floating objects in still water. The object in question Figure (1) is (0.1x0.12x0.01m) in x, y and z directions and at very low densities it will remain with its long axis vertical. At some density (137 kg/m^3), the float is no longer in stable equilibrium with its long axis vertical, and will list in the water at some angle theta 'θ', at which it will then be stable. This angle has been calculated for increasing densities of the object, and the relationship between stable angle and density is plotted below:

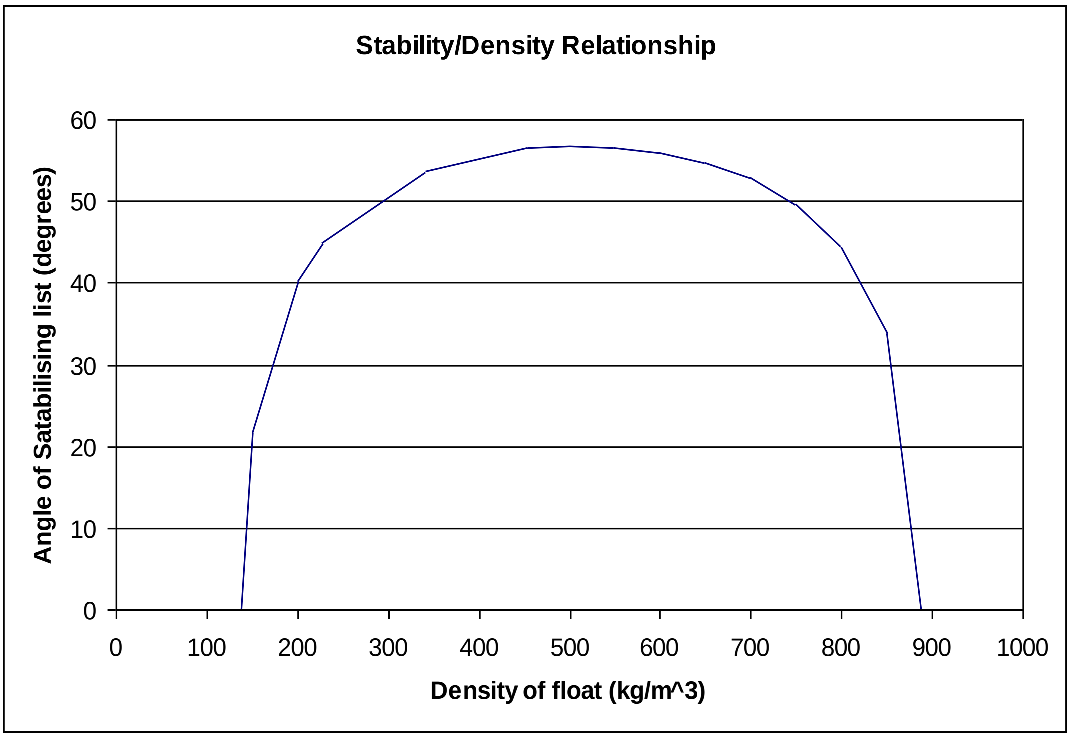


Figure 4

Stability position of floating box

The graph shows that at extremes of density the angle returns to zero, whereas for intermediate values of density, the angle of stability increases to a maximum of 57 degrees when the density of the float is half that of the water.

For the cases of stability of (Table 1) where the density is low compared with that of the water, the block is theoretically stable. Its geometry is such that it starts with its longer axis vertical, and statics dictates that it should remain so. In reality this configuration is actually highly unstable as it is analogous to a 'tall' block resting on an extremely flexible foundation. The block would be expected to tilt over and come to 'rest' in a position with its long axis horizontal, i.e. the centre of gravity ends up in its lowest position to minimise the potential energy within the rigid body

Note: this particular problem is related purely to geometry of the structure.

Simple but efficient criteria is given based on statics and Archimedes principle, a graphic representation and table and curve linking angle of list, righting moment to density are illustrated as criteria to monitor stability of floats. The method discussed "one of many methods widely reported in literature" might be necessary but not sufficient for all floating objects specially if hydrodynamic loads were involved, the moving centre of gravity for floaters carrying fluids or grained materials will definitely limit the use of this criterion. floating object stability is essential to use in the above defined ALE feature used in explicit codes for finite element analysis.

Table 1

Table 1 Stability positions for buoyant small 2-D box

Case	h m	γ kg/m^3	$\Theta_{theoritical}$ degree	metacentric height mm	stability condition
1	0.010	087.980	0.00	26.055	stable
2	0.013	113.600	0.00	9.3090	stable
3	0.016	136.900	0.00	0.0080	stable
4	0.027	228.070	44.9	-15.440	unstable
5	0.040	341.670	53.7	-19.167	unstable
6	0.053	455.271	56.5	-17.715	unstable
7	0.067	569.729	56.5	-14.156	unstable
8	0.080	683.333	53.7	-9.5830	unstable
9	0.093	796.938	44.9	-4.4180	unstable
10	0.100	854.170	32.4	-1.6670	unstable
11	0.104	888.100	0.00	0.00100	stable
12	0.105	900.000	0.00	0.59100	stable

References

[1] Vugts J h et al "Design of the Support Structure of an Optimised Offshore Wind Energy Converter" Proc. EWEC' 99 (1999).

[2] Swift R H & Dixon J C 1982, "Design Wave Forces on Offshore Structures", BWEA 4th wind energy conference Cranfield UK.

[3] Patel M H "Dynamics of Offshore Structures" Butterworth 1989.

[4] Clause G F and Birk L "Design of Optimum Structures Based on Long Term Wave Statistics" 17th OMAE Conference 1998, paper 0521.

[5] Mohamed A Almherigh, "Evaluation of Finite Element Techniques Applied to Floating Offshore Wind Energy Turbine", Ph. D thesis, The University of Salford, Salford, UK, 2006.